Principles of Soil Mechanics:
II—Compressive Strength of Clay

Modulus of Elasticity Determined from Cube Tests at Different Moisture Contents Shows Constant Ratio to Capillary Pressure—Poisson's Ratio for Clay—Analogies with Metals—The Iowa Experiments

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FOLLOWING the tests of confined samples of clay reported in the preceding article (*Engineering News-Record*, Nov. 5, 1925, p. 742), cubes of clay were tested by methods the same as used for ordinary solids. These tests brought somewhat different phenomena into view. In particular they threw light on resaturation action, on lateral effects, and on the analogies between clay and solids. The careful attention paid to the elastic properties of clay cubes was warranted by the fact that the behavior of clay cubes under load was found to be closely related to what is called the bearing capacity of clay deposits. As a consequence a clear understanding of the effect of load on clay cubes represents the key for understanding the more complicated effect of a load placed on the surface of a clay deposit.

Elastic Behavior of Clay

Cubes 2 cm. and 4 cm. on a side were used. They were molded within a lining of filter paper in a prismatic mold fitted with a plunger. According to the purpose of the test, the cube was either tested at once after molding or it was allowed to dry until the moisture content decreased to that desired. During the test the cube was surrounded by a tin cylinder inclosing an inner one of brass wire mesh, with watersoaked cotton between, to maintain an atmosphere saturated with water vapor. In such an inclosure the moisture content of the clay remained sensibly constant for several days, while a test lasted only two hours.

To measure the compressive shortening of the cubes measurements of great precision were required, and the accuracy of a micrometer screw with electric contact indicator proved to be insufficient. The difficulty was overcome by developing an interference contact indicator, consisting of a pair of thin glass sheets with a residual film of water between, a very sensitive micrometer screw, and an eyepiece. By noting the change in color of the Newtonian rings due to contact between the upper glass sheet (thickness 0.1 to 0.2 mm.) and the point of the micrometer screw, the position of the screw point can be determined to less than 0.00001 mm., which is far beyond the degree of accuracy of the micrometer screw.

Cube Test Results—The main diagrams in Fig. 1 may serve as an example of the results obtained by the tests. They represent the stress-strain diagrams for a set of cubes of yellow residual pottery clay with different percentages of moisture. The diagrams are very similar to compression diagrams of concrete and natural stone. Abscissas represent pressures (kg. / cm.^{2}), while ordinates give reduced strain, i.e., the ratio between the total compression and the reduced height—the height which the cubes would have if the volume of voids were reduced to zero at constant horizontal cross-section. Operating with reduced instead of ordinary strains is necessary in order to have a common basis for comparing with each other the strains produced by loads in cubes of different moisture content.

Strain Lag—In testing Cubes 1 and 2 the load was first gradually increased and then left constant for a certain time. During this time the strain increased, at decreasing rate (strain-time curves, under the stress-strain diagrams). This change of strain at constant load is unlike the change of moisture content at constant load in the tests previously described, as the latter process is due to the great resistance against the flow of water through the voids of the clay, while the former one goes on at constant moisture content, i.e., with the capillary water remaining in a state of equilibrium. The increase of the strain of clay cubes at a constant load seems to be identical with the elastic after-effects associated with the deformation of solid elastic bodies. The author has found that in clay it is due to the gradual compensation of unbalanced frictional resistances which at first take up a part of the load. Simple mathematical relations exist between the capillary pressure acting within the cube, the load and the value of the coefficients which determine the relation between the time and the increase of the strain.[1]

Cubes 3 and 4 in Fig. 1 were tested on a screw testing machine, in which the compression (strain) remains constant when the machine is stopped, while the pressure may change. As the lower diagrams show, when the compression is thus held constant the pressure decreases for a time. This phenomenon was found to be due to the same cause as the increase of strain at constant load, and it follows similar simple laws.

The more slowly the load was applied, the less marked were the elastic after-effects, but at the same time the steeper was the slope of the main branch of the stress-strain diagram. The main branch, which corresponds to infinitely slow application of the load, is called the reduced stress-strain curve (dash-line curves in Fig. 1). It can easily be constructed by means of the data furnished by the time-strain or the time-pressure curves.

Cyclic Loading—The next operation performed during the cube tests consisted in reducing the load to zero and re-applying it (complete cycle). The effect is represented by the hysteresis-loops. These loops differ from those shown in the pressure-moisture diagrams previously described by being straight instead of curved. When the load again reaches the value it had before release, the stress-strain curve asymptotically approaches the curve of continuous increase of load. Phenomena similar to these came in evidence when plotting the results of loading tests performed on the bottom of test pits in the field.

Modulus of Elasticity—The term "modulus of elasticity" should be confined to the reversible part of the deformation of the cube, represented by the hysteresis loops. It is equal to the tangent of the angle between the strain axis and the axis of the hysteresis loop. In general this angle differs but very little from that of the initial tangent to the continuous-loading curve.

A very important relation was found to exist between the intensity of the capillary pressure and the value of the modulus of elasticity. The abscissas of the main branches of the pressure-moisture curves (see Fig. 3, p. 743, *Engineering News-Record*, Nov. 5, 1925) indicate the pressures required to reduce the moisture content of confined clay from its initial value (for zero pressure) down to the moisture content represented by the ordinates, provided surface tension is not active. It has been shown that the capillary pressure acting within a clay of definite moisture content is equal to the external pressure which in the pressure-moisture diagram corresponds to that moisture content. Using these pressure equivalents of the moisture content the curves in Fig. 2 were plotted. Here abscissas represent capillary pressures and ordinates represent moduli of elasticity. The values for Cubes 1, 2, 3 and a fourth cube intermediate in moisture content between 2 and 3 are plotted on the line marked "Yellow Pottery Clay." As the points mark out a straight line, it appears that the ratio between modulus of elasticity and capillary pressure of a given kind of clay is constant and independent of moisture content, provided that no resaturation of the sample has occurred. The lower line is the corresponding curve for a blue marine clay and the two upper lines for two sands. The simple relation which exists between the capillary pressure and the modulus of elasticity is of considerable practical importance inasmuch as it was found that the bearing capacity of a clay increases in simple proportion with its modulus of elasticity.

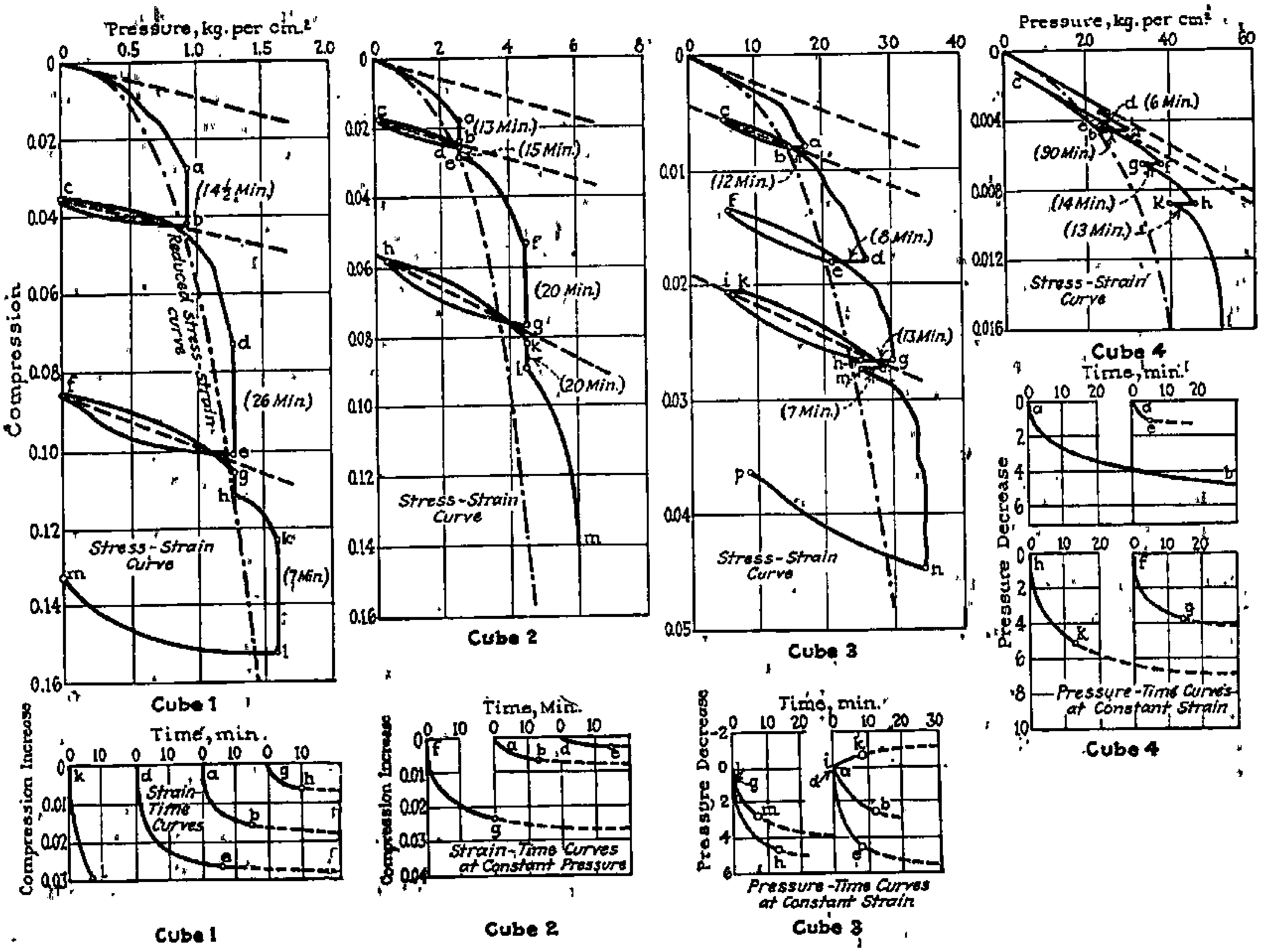

FIG. 1—COMPRESSION TESTS OF CLAY CUBES

Yellow residual pottery clay, specific gravity of solid matter 2.93, lower limit of plastic state 24.2, lower limit of liquid state 58.0 per cent, coefficient of plasticity 33.8.

	Cube 1	Cube 2	Cube 3	Cube 4
Moisture-Ratio e, (per cent of volume of solid matter)	0.792	0.681	0.490	0.482
Modulus of Elasticity (kg. per cm².)	115 to 76	310 to 195	3,760 to 3,460	7,300

Analogy of Clay and Solids—Cohesion of clay depends upon the capillary pressure. In a similar way the cohesion of solid bodies is the result of the "intrinsic pressure," i.e., of the pressure per unit of area produced by the mutual attraction of the molecules. As the elastic properties of the clay cubes are very similar to those of solid cubes, the author suspected that the ratios between the moduli of elasticity of solids and their intrinsic pressures might also be a constant. This conclusion was found to be correct. In the upper diagram in Fig. 2, abscissas represent the intrinsic pressures of various metals (according to Traube, computed from thermodynamic data), while ordinates are moduli of elasticity for compression within rigid enclosures, computed from the ordinary moduli of elasticity and the

Poisson constants. All the points are located along a straight line.

If the moduli of elasticity were perfectly constant for each clay cube, the hysteresis loops ought to be strictly parallel to each other, while in the diagrams Fig. 1 the slopes of the loops somewhat increase with the intensity of the pressure at which the cycle was started. In order to understand this we must keep in mind the fact that in case of a solid the intrinsic pressure remains constant during a compression test, while the volume decreases. It would be possible to maintain the volume constant by reducing the intrinsic pressure as the load is increased. In the case of a clay cube the volume cannot possibly change as long as the moisture content is unchanged, and therefore the capillary pressure decreases with increasing load, which in turn causes a decrease of the modulus of elasticity and a corresponding increase of the slope of the hysteresis loops. As a matter of fact, after having determined Poisson's ratio for several clays, the author was able to compute the relative position of their hysteresis loops, and the results checked fairly closely with the results of tests.

Poisson's Ratio; Resaturation—Poisson's ratio represents the ratio between the linear compression and the corresponding linear lateral expansion for a loaded cube. If one knows this ratio for any homogeneous material one can calculate the lateral pressure which the material will exert under load against a rigid enclosure. In turn, knowing the lateral pressure, one can compute Poisson's ratio. This method was used for the clays.

In order to find the lateral pressure exerted by a loaded layer of clay (mixture of clay and water, free from air and the clay surfaces covered with water so as to exclude the surface tension of the water), two sets of apparatus were used similar to the one represented by Fig. 2 (p. 743, *Engineering News-Record*, Nov. 5, 1925). The rings were larger and higher, and the clay was enclosed between two filters of equal dimensions and quality. At half the height of each layer of clay there was a horizontal steel tape, which passed out through slits in the enclosing ring. In one of the layers the flat side of the tape was horizontal, in the other one it was vertical. In order to keep the clay from being squeezed out through the slits, each slit was sealed on the inside by a small shield of thick filter paper. The ratio K between the horizontal and the vertical pressure acting within the clay is equal to the ratio between the forces required to overcome the friction between the clay and the two tapes. The frictional resistances were not measured until three or more days after the load was applied, to allow the hydrostatic stress differences to disappear. Computation from the results furnished the values of Poisson's ratio:

Yellow residual clay ($K = 0.70$)	2.73
Blue marine clay ($K = 0.75$)	2.56
(Water	2.00)
(Lead	2.24)
(Silver	2.63)

If both Poisson's ratio and the ratio between modulus of elasticity and capillary pressure are known, one can compute the reversible part of the deformation produced by a load on a layer of clay confined within a rigid ring. This reversible part is represented by the curved hysteresis loops of the pressure-moisture diagram (Fig. 3, p. 743, *Engineering News-Record*, Nov. 5, 1925). The computation was carried out for different kinds of clays. In every case the curve thus obtained coincided very nearly with the recurrent branches of the hysteresis loops of the corresponding pressure-moisture diagrams. This furnished a check on the computation of Poisson's ratio.

The recurrent branches of the hysteresis loops of the pressure-moisture-content diagrams are called the *resaturation curves*. They are simple logarithmic lines. The greater the resaturation coefficient, the greater is the increase in volume due to resaturation and the smaller is the ratio between modulus of elasticity and capillary pressure. The coefficient is greatest for clays rich in colloids. Experience seems to show that a clay is the more likely to slide the more it swells when brought into contact with water. Hence the value of the coefficient of resaturation may furnish a valuable indication of the stability of a natural clay deposit.

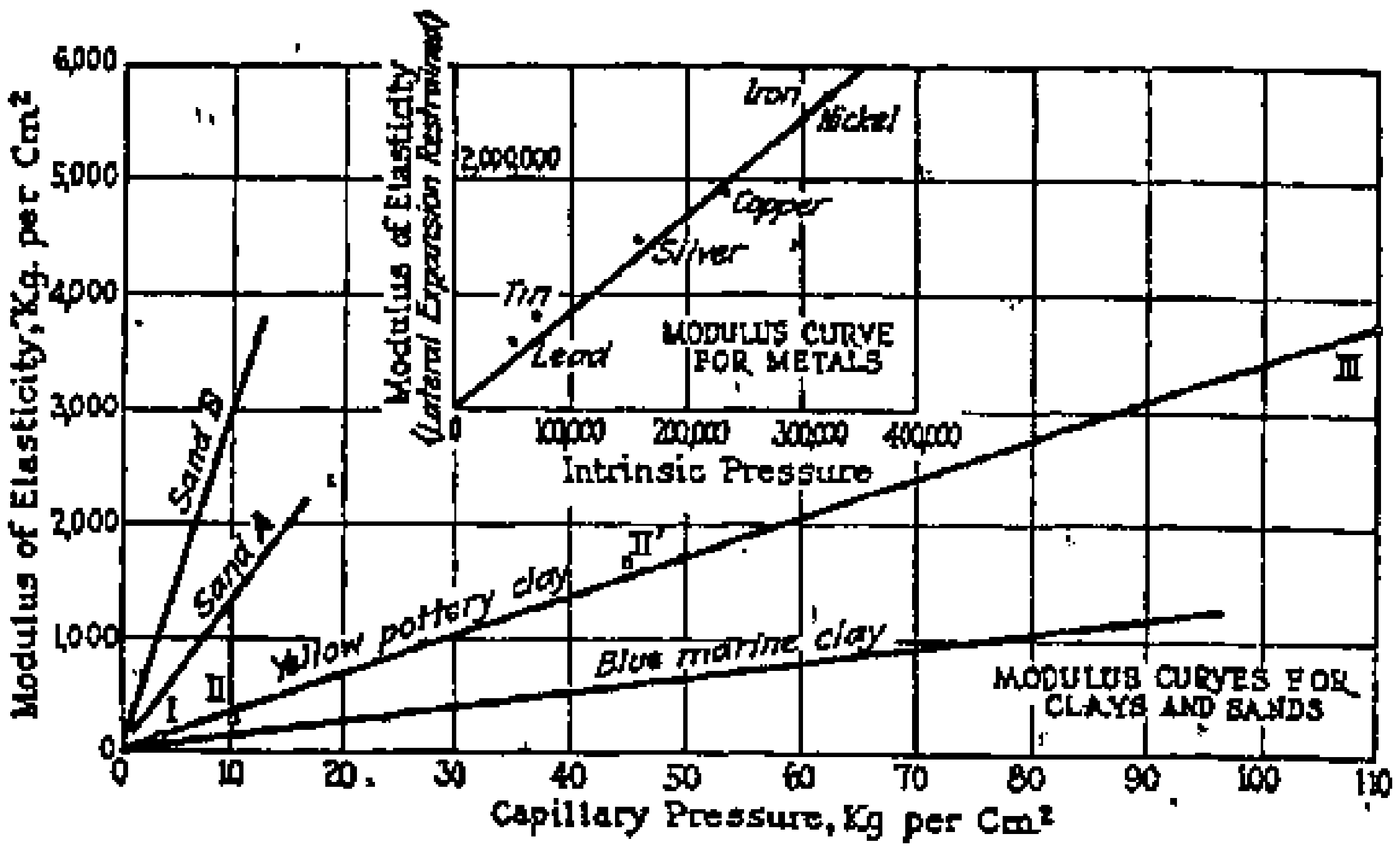

FIG. 2—RELATION BETWEEN MODULUS OF ELASTICITY AND CAPILLARY PRESSURE

Non-homogeneous Clay Soils—In 1921 and 1922 the engineering experiment station of Iowa State College determined the compressive, tensile and shearing strength of different kinds of clay exposed on the bottom of an old gravel pit. The samples were taken from a depth of 1.5 to 1.8 m. below the bottom of the pit and were tested in undisturbed condition. The stress-strain diagrams seem to resemble closely those presented in Fig 1. Nevertheless, a thorough study of these diagrams leads to the conclusion that the two sets of tests differ in vital respects.

For the Iowa cubes the reversible part of the deformation is very small compared with the total deformation, and the high moduli of elasticity of the cubes seem quite out of proportion to their low compressive strengths. Calculation indicates unusually high coefficients of internal friction. One of the causes of these abnormalities seems to reside in the fact that the Iowa clay apparently had undergone repeated resaturation and shrinkage since its deposition, while the cubes of Fig. 1 were compacted under the influence of direct compression by capillary pressure. According to our experience, resaturation substantially affects the relation which exists between the moisture content and the capillary pressure. However, up to the present time, no exhaustive investigation of the effect of resaturation has yet been made.

The second and equally important cause of difference lies in the fact that the cubes of Fig. 1 were homogeneous, while the Iowa cubes were not. Thus, the paper states. "The yellow clay . . . was not infrequently permeated by roots, wormholes and fibers or small crevices, and the strength would be correspondingly decreased." "One of the prominent factors appeared to be the friability or brittleness of the blue clay, as induced by the infiltration of water and air previously mentioned, which oxidizes the ferrous matter and causes the deposition of a brownish-yellow film along the planes of separation which honeycombed the structure. A slight shock as a result of this film or matrix was often sufficient to leave the blue clay a coarse granular mass." (p. 563). Clays which include planes of separation and seams of oxidized matter cannot be considered homogeneous; they represent an intermediate type between homogeneous clay and the crumb soils. In a homogeneous clay, the hydrostatic pressure of the capillary water is the same throughout the mass (provided the clay is in hydrostatic equilibrium); in a soil composed of crumbs the particles of each crumb are bound together by a very intense capillary pressure, while the crumbs themselves are as independent as are the individual grains of a mass of sand. As a matter of fact, accumulations of dry crumbs prove to have the same elastic properties as dry sands except for the fact that they are more compressible and less elastic than sands, on account of the brittleness of the crumbs. If water penetrates the voids of such a crumb mass, the crumbs absorb the water very slowly, because their permeability is small. Water flows from the voids between the crumbs towards the centers of important negative hydrostatic pressure, i. e. penetrates the crumbs. However, even after equilibrium is reached the soil represents a honeycombed mass which can by no means be compared to a homogeneous clay and if the water later on evaporates, the mass breaks up into crumbs again, on account of the non-uniform resistance of the material against the effect of secondary stresses.

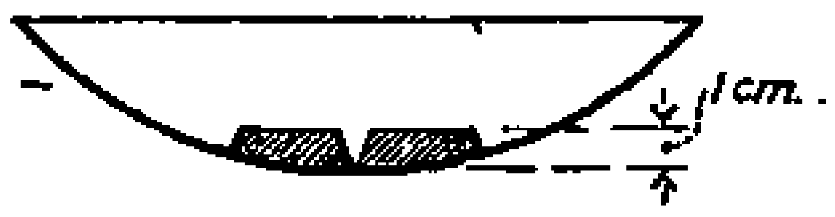

FIG. 3—DETERMINING LIMIT OF LIQUID STATE (ATTERBERG)

Due to these facts, there is a great difference between the elastic properties of crumb soils and of homogeneous clays. The former still need thorough experimental investigation. Fortunately the worst types of foundations (mud deposits, soft clays and the like) belong almost without exception to the homogeneous type of clay, whose physical properties are now known.

It is to be regretted that the Iowa investigations could not be supplemented by contemporaneous loading tests performed with the soil-testing apparatus proposed by the Foundation Committee.[3] The results would have furnished a most valuable contribution to our knowledge of the resistance of clays, because the relation which exists between the cube strength of a resaturated clay and the bearing capacity of the same material is not yet known.

Data Required for Describing Clays—Engineering literature contains hardly a single description of a clay that would allow us to identify the clay with those from other localities, or a description which gives us a clear notion of what the material was like. At best the describer contents himself with mentioning the color of the clay, its moisture content and its chemical composition. Experience, however, has shown that the chemical composition of a clay has but little to do with its physical properties, and, as for moisture content, some clays with 25 per cent of water are almost liquid while others containing 30 per cent are very stiff.

An exhaustive investigation of test boring samples requires too much time and labor to be justified except for scientific purposes. Study of the problem of investigating the properties of clays for *engineering purposes* has led the author to the conclusion that it would be advisable to modify the test program according to the character of the work affected. A detailed account of the data required for describing clays extracted from the sites of future floating or pile foundations, clays for dam construction purposes, clays which threaten to slide, etc., has already been published (in the author's book "Erdbaumechanik," Vienna, 1925). Some data however are required regardless of the special purpose of the description.[4] These data concern the relation between consistency and moisture content.

The simplest and most reliable way of expressing this relation was devised by the late Prof. A. Atterberg, of Kalmar (Sweden), for agrogeological purposes, but it suits engineering purposes as well.[4] It requires a knowledge of the following data: Moisture content of the clay, specific gravity of the dry matter, lower limit of the plastic state and lower limit of the liquid state.

Moisture content is determined in the ordinary way. The sample is taken out of the test-pit or out of the well-boring auger and transferred into a wide-necked bottle with glass stopper. The bottle is filled completely so that there remains no air between the clay and the stopper, and the joint between the neck and the stopper is sealed with wax. In the laboratory a sample of the content is placed between two watchglasses and weighed, dried at 100 deg. C. and again weighed.

The specific gravity of the dry matter has to be determined by pycnometer. In order to get fairly accurate results the clay solution should be heated to the boiling point to drive out the adsorbed air. The test is made after the solution has cooled.

The lower limit of the plastic state is determined as follows: Mix a sample of the clay with water until it becomes very plastic or, if it is too soft in its natural state, let it dry until it reaches the consistency required. Then work the sample into several thin threads (diameter about 3 mm.), put the threads together and work them out to threads again by rolling them with the palm on a smooth, clean sheet of paper. Repeat the process until no more threads can be formed, the material becoming brittle and the threads breaking to pieces while worked. Then the moisture content is determined; this value, expressed in per cent of the weight of the dried sample, was called by Atterberg the *lower limit of the plastic state.*

In order to determine the lower limit of the liquid state, take a flat porcelain cup (Fig. 3), mix a sample of the clay with water until it becomes very soft, make out of the mixture a cake about 4 cm. in diameter and 0.8 cm. thick, cut this cake with a nickel scoop into two equal parts and shake the cup. If the two lower edges of the cut do not flow together, add some water and repeat the test. The moisture content at which the lower rims join along a strip of a height of about 1 mm. is called the *lower limit of the liquid state.* Both limits should be determined at least twice.

The difference between the two limits is called the *coefficient of plasticity.* A clay is plastic between the two limits, else it is either liquid or semi-solid. The greater the coefficient of plasticity, the more plastic the clay is supposed to be. For materials without any plasticity (typical quicksands or very fine quartz dust) the coefficient is zero, i.e. the two limits are identical. For equal coefficients of plasticity the limit of plasticity may be low or high, depending on the shape of the grains and the percentage of humus constituents present.

The degree of plasticity of two clays with equal coefficients of plasticity may be equal or different according to whether the specific gravities of their grains are equal or different. In order to establish a common basis for comparison, the author expresses the limiting moisture contents in per cent of the space occupied by the solid matter. Care should be taken to use samples which have not previously been dried, as the very great capillary pressures which develop during the process of shrinkage may crush many of the grains and decrease the average size of grains.

The data mentioned represent the minimum requirement for characterizing a clay. Without containing these data, the description of a clay is practically worthless and the recorded phenomena are merely curiosities.

Summary—From the experimental studies described in this and the preceding article we have derived facts which may be summarized as follows:

The cohesion of clay is due to two factors. One of these is the pressure exerted by the surface tension of the capillary water, a force whose intensity exceeds all the other forces the earthwork engineer has to deal with. It may amount to several hundred atmospheres: it compacts loose, colloidal sediments more thoroughly than can be done by artificial means except in the laboratory by using a high-power testing machine. Swelling of clay is nothing more or less than the purely elastic expansion produced by the elimination of the surface tension of the capillary water. Local evaporation of the capillary water or local flooding of the surface of clay deposits produces secondary stresses the intensity of which is far greater than the weight of the heaviest structures and which were found to be the primary cause of many vast soil displacements, known as *earth slips.*

The second one of the factors mentioned consists in the fact that the properties of the water contained in voids of width less than 0.0001 mm. are no longer identical with those of ordinary water. In such voids, viscosity and surface tension are increased (in inverse proportion to the diameter of the voids), and the water loses its ability to evaporate in contact with the air. Thus the capillary water of the clays is to a certain degree solidified by the influence of the forces exerted by the molecules of the solid matter. Due to this fact the capillary pressure assumes far greater values than it would if the surface tension of the capillary water had its normal value.

Capillary pressure plays the same part in the physics of clays as does intrinsic pressure in the physics of solids. Therefore the elastic properties of the clays are qualitatively identical with those of granular solids (rocks, concrete, etc.).

The minimum requirement for describing a clay consists in presenting the following data: Water content, specific gravity of the solid matter, lower limit of the plastic and the lower limit of the liquid state of the clay.

References.

[11] "Die Beziehungen zwischen Elastizität und Innendruck," K. Terzaghi, *Sitzungsberichte der Wiener Akademie der Wissenschaften*, 1923.

[12] "Progress report of tests on undisturbed clays," J. W. Griffith, *Proceedings, Am. Soc. C. E.*, March, 1922, pp. 557-579.

[13] *Proceedings*, Am. Soc. C. E., August 1920, Plate XI and XII; January, 1922, Plate VII.

[14] "Die Plastizität der Tone," A. Atterberg, *Internationale Mitteilungen für Bodenkunde*, 1911. Atterberg's method of measuring plasticity was severely criticized by W. E. Emley five years ago (Technologic Paper 169, U. S. Bureau of Standards, "Measurement of Plasticity of Mortars and Plasters"). The arguments advanced by Mr. Emley are justified so far as the plasticity of mortars and plasters for industrial uses is concerned. But they do not seem to affect the value of the method for purposes of soil study.