

# Principles of Soil Mechanics:

## III—Determination of Permeability of Clay

Validity of Darcy's Law—Slichter's Formula and Hazen's Observations—Permeability of Sand—Experiments on Clay—Darcy's Law Valid Even for Semi-Solid State

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WHEN water percolates through a permeable body of prismatic form in parallel flow, its rate of flow is expressed by Darcy's formula,  $Q = kFs$ , where  $Q$  is the volume of flow per second,  $F$  is the cross-section of the body, and  $s$  is the hydraulic gradient, equal to loss of head divided by length, or  $h/l$ . The coefficient  $k$  (dimensions cm./sec.) has been called the *coefficient of permeability*, and represents the velocity of percolation under a hydraulic gradient of unity.

The law is valid for laminar flow only, and therefore does not apply to the (turbulent) flow of water through coarse gravel. Some have claimed also that it is not valid for flow through clay. But it is generally accepted as applicable to flow through fine and medium sand under moderate head, as it agrees fairly well with experimental data.

**Void-Ratio and Permeability**—The voids of a permeable body constitute in effect a system of capillary tubes. Percolation velocity depends on the size of the tubes. A sand of definite grain size may have wide or narrow voids, according to whether the structure is loose or dense, and the coefficient of permeability thus depends on density of structure as well as on grain size.

Many years ago Prof. C. S. Slichter<sup>1</sup> made an attempt to compute the coefficient of permeability of sand, assuming uniform globular grains; according to the arrangement of the spheres, the volume of voids ranges between limits of 26.0 and 47.6 per cent of the total volume of the mass. His formula was  $k = 771d^2/c$ , where  $d$  is the diameter of the spheres in centimeters and the factor  $c$  has for a temperature of 10 deg. C. values as follows:

Volume of Voids = $\frac{\text{Voids}}{\text{Total Volume}}$ =							
$n$ ,	0.26	0.28	0.30	0.34	0.38	0.42	0.46
Void-Ratio = $\frac{\text{Voids}}{\text{Solid Volume}}$ =							
$e$ ,	0.352	0.388	0.428	0.515	0.612	0.728	0.850
Factor $c$ ,	84.30	65.90	52.50	34.70	24.10	17.30	12.80

Thus, the coefficient of permeability of a mass with a volume of voids of 46 per cent (very loose arrangement of the spheres) becomes  $60.3d^2$  at 10 deg. C. But Allen Hazen has found by experiment that the coefficient of permeability of a very clean and loose sand amounts to  $116d_w^2$  ( $d_w$  is effective size of the grains according to his well-known definition), and, if the grains are of approximately the same size, even  $150d^2$ .

The disagreement between the theoretical value obtained by the formula of Slichter and Hazen's experimental results is due to the fact that the voids of Slichter's filter are supposed to be equal among themselves, while in practice each filter contains wide and narrow voids. It can be proved that at equal volume of voids and equal size of grains the coefficient of permeability decreases with increasing uniformity of voids.

On the other hand, the relation which exists for any one kind of sand between the density and coefficient of permeability agrees fairly well with Slichter's theory.

In order to represent the theoretical relation, the void-permeability curve  $S$  in Fig. 1 has been plotted. It corresponds to a mass consisting of microscopically small spheres of equal diameters. The upper and lower ends of the middle section of the curve (void-volume 47.6 per cent, void-ratio 0.905; void-volume 26 per cent, void-ratio 0.352) indicate the extreme limits within which Slichter's formula is valid. The geometrical part of the theory of Slichter has been further developed by L. Darapsky and L. Burmester<sup>2</sup>; but, as the shape of the grains of natural sands is irregular rather than globular, elaborate theoretical investigations of such a kind are at best of academic value.

**Permeability of Sand**—As Slichter's formula for the permeability coefficient is rather complicated, I derived a semi-empirical formula based on the following facts: The widest parts of the capillary channels through which the water flows have at least five times the cross-section of the narrowest ones. Hence, if a definite quantity of water percolates through one of the capillary channels, the loss of head per unit of length of the narrowest sections of the channel is at least 25 times greater than the loss per unit of length of the widest ones. Due to this, the percolation of water through sand can be compared to the flow of water through a set of sieves in series; the resistance to percolation is confined to the sieves while in the spaces between sieves the resistance is negligible. Let  $d_w$  be the effective size of the grains (cm.),  $n$  the void-volume,  $v_0$  and  $v_t$  the coefficients of viscosity of the water at 10 deg. C. and at temperature  $t$  respectively, and  $C$  an empirical coefficient found by experiment to range from  $800v_0$  to  $460v_0$ , depending both on the shape of the grains and on the uniformity of the sand; then

$$k = \left(\frac{C}{v_0}\right) \left(\frac{v_0}{v_t}\right) \left(\frac{n-0.13}{1-n}\right)^2 d^2$$

$$= (800 \text{ to } 460) \frac{v_0}{v_t} \left(\frac{n-0.13}{1-n}\right)^2 d^2$$

The value 800 was derived from tests on sands whose grains were well polished and rounded, while the value 460 was from tests with sands of irregular, rough grains. The influence of the uniformity of the sand upon the value of  $C$  was far less marked than the influence of the quality of the grains. Within the limits of void-ratio 0.352 and 0.905 (void-volume 26.0 to 47.6 per cent) the curve of permeability, on void-ratio plotted by means of the formula just given, coincides closely with the corresponding curve plotted from Slichter's formula (Fig. 1), provided they are drawn so as to have one point in common. It may surprise the reader

to learn that two apparently very different methods for calculating the relation between the volume of voids and the coefficient of permeability furnish similar results. But it has been found by experience that any simplifying assumptions concerning the shape and the arrangement of the capillary openings lead to results agreeing fairly well with each other and with the observations, provided the nature of the material has been taken into consideration appropriately. This fact considerably facilitates the interpretation of the results of tests.

Basing calculations on the formula, one can reduce any coefficient of permeability to a volume of voids of 50 per cent (void-ratio 1.0) and to a normal temperature of 10 deg. C., regardless of the actual density of structure of the sample. The coefficient of permeability thus obtained is called the *reduced coefficient of permeability*  $k_r$ . By introducing into the formula the special values  $n = 0.5$  and  $v_o = v_t$ , one arrives at  $k_r = (174 \text{ to } 100) d_w^2$ . As the value  $k_r$  is independent of both temperature and voids, it is a value which may serve as a basis for investigating the influence of the quality of the grains and of the uniformity of the mass upon the permeability of the material. The values 174 and 100 seem thus far to represent the extreme limits for these influences, provided the sands are perfectly clean, without any traces of clay. Even a very small quantity of clay was found to upset the validity of the formula. For checking the formula obtained by theory, and for determining the value of the empirical coefficients, the relation between void-ratio and coefficient of permeability had to be investigated by experiment.

**Tests of Permeability of Sand**—In order to get accurate results care must be taken that neither the sand nor the water system contains air. I use a cylindrical filter (Fig. 2), the sand resting on a diaphragm of brass wire mesh. Provision is made for sending water through the sand either upward or downward. Before the sand is introduced the water is allowed to rise above the wire mesh, and bubbles adhering to the mesh are removed by exhaustion. The sand is thoroughly wetted before it is placed in the cylinder. If, during the test the water flows upward, the rim of the cylinder serves as overflow. However, most of the tests were made with the water percolating downward. The loss of head is measured by a graduated standpipe, communicating with the space below the wire mesh. The data required for making a complete report on the results of the test are surface area and thickness of the layer, method of preparing the layer (sand loosely poured in, stirred, compacted) volume of voids, average specific gravity of the grains, temperature, sieve analysis, effective size of grains, shape of grains, values of percolation and loss of head. Basing on the results of numerous tests, the formula expressing the relation between void-ratio and permeability has been checked and verified.

**Permeability of Clay**—If the grains of clay differed from those of sand only in size, our formula could be used for computing the coefficient of permeability of both pure sand and pure clay. But it was found that the finer constituents of clay consist of very thin, flexible mineral scales, while the grains of sand are bulky and rigid. The capillary channels of sand accordingly have a rather compact cross-section, while those of clay resemble narrow slits. This difference has an important influence on the mechanics of the percolation proc-

ess. A theoretical study of the flow of water through powders with scale-like grains led to the formula

$$k = \left(\frac{C}{v_o}\right) \left(\frac{v_o}{v_t}\right) (e - 0.15)^2 (1 + e)^2 d^2$$

where  $e$  is the void-ratio, or ratio of volume of voids to volume of solid matter. Considering the verified accuracy of the formula for sand, the present formula might also be expected to be adequate. However, before empirical data for verifying the formula could be obtained, it was necessary to develop a method for determining experimentally the coefficient of permeability of clay.

**Clay Tests**—Measuring the permeability of clay is more difficult. The first systematic attempts seem to have been made by American engineers (testing materials for the Lahontan dam, Cold Springs dam, Palouse project). In most of these tests the material was put into a cylindrical vessel and compacted by tamping. The height of the layer was usually about 100 cm. Below the layer of soil there was a filter. The water percolated vertically downward. In principle the apparatus was identical with that shown in Fig. 2, and it would fully serve for determining the coefficient of sand. But when applied to clay the method is open to serious objection.

Experience has shown that the coefficient of permeability of clay varies within extraordinarily wide limits, according to the volume of voids. As soon as a soil mass composed of crumbs of clay comes into contact with water, the crumbs swell and the volume of voids increases. Within the test cylinder this swelling is restrained to an unknown extent by friction along the sides of the vessel and it is doubtful if the volume of voids of the sample is identical with that of the soil in the dam. Furthermore, the samples unavoidably contain much air. Even if one shakes a sample of powdered clay with water in a bottle, a considerable quantity of air continues to stick to the clay particles and one cannot drive it out except by boiling the solution. But in the dam the air contained in the soil is gradually carried away by the percolating water in solution, while the laboratory test never lasts long enough even partially to exhaust the air out of a sample 100 cm. thick. Last but not least, one must consider that a crumb soil, as used by the investigators, contains voids much larger than the coarsest individual particles of the soil. Water percolating through such a mass carries in suspension the fine particles detached from the crumbs located near the upper surface of the sample, and the formation of a "filter skin" becomes inevitable. As soon as a filter skin is formed, no more conclusions can be drawn from the results of the tests regarding the permeability of the samples. It is very doubtful whether the coefficient of permeability of clay crumb soils can be determined at all except by indirect experimental methods.

In order to exclude these sources of error I constructed the apparatus shown in Fig. 3. Air can easily be excluded from the interior of the apparatus; the diameter of the clay sample is much greater than its thickness, hence the friction along the sides of the sample has practically no influence on the volume of voids; and the moisture content of the sample is maintained constant by keeping the sample under a constant pressure.

The apparatus consists of a vessel in which an internal rim rests on a perforated bronze plate, whose upper side is covered with a fine brass wire mesh. The plate

supports a sand filter covered with a circular sheet of thin filter paper. In order to perform the test, one fills the vessel with water, introduces the bronze plate, removes the air bubbles from the wire mesh, fills the space above the mesh with clean quartz sand (size of grains 0.5 mm.), and covers the surface of the filter with filter paper. In the meantime the ring shown seated on the vessel has been partially filled with a plastic, homogeneous mixture of water and clay, about 2.5 cm. thick. This ring is applied to the top of the vessel in such a way that the air may escape when the ring is lowered, and is then clamped down by a gasketed bronze ring. The upper surface of the clay sample is covered with paper and a sand filter. This is loaded by a lever. The water-filled space below the perforated bronze plate communicates with a standpipe 100 cm. high, 0.6 cm. in diameter.

Under the influence of the load on the clay, the moisture content of the sample decreases, until equilibrium is reached. Then one fills the standpipe with water and the water percolates upward through the clay. As a rule more water evaporates in the small annular space around the loading plate than comes up through the clay. Hence it is necessary to keep the water-level in the annular space constant by occasionally adding water. In the standpipe the water-level falls during the test at a decreasing rate. I make readings three times a day and compute the coefficient of permeability from the rate at which the level sinks.

Three tests are run simultaneously, at loads of 60, 125 and 190 kg. (0.75, 1.6 and 2.4 kg./cm.<sup>2</sup>). As a rule four to six weeks are required for the contents of the standpipe to filter through a fairly fat clay. After each reading, the temperature of the water is read from a thermometer whose bulb is enclosed within a brass pipe (not shown on the drawing) which passes across the vessel below the perforated bronze plate. As changes of temperature have marked effect on the elevation of the water level in the standpipe, the tests must be made in a cellar with a very constant temperature. A fourth simultaneous test is run in an apparatus of somewhat different type, containing a sample at about the lower limit of the liquid state, not loaded, the water filtering through the clay in downward direction from a standpipe over an opening in a bronze plate clamped down on the gasketed upper rim of the cylinder which contains the clay. After the test the clay in each apparatus is sampled and its moisture content determined.

Homogeneous mixtures of clay and water were used only. As the width of the voids of such samples is equal to or less than the diameter of the smaller individual particles, no filter skin can be formed, provided the water used for the test is perfectly clean.

In order to investigate whether a filter skin forms in these tests, parallel tests were run with samples of different thickness. If a filter skin formed, a thin layer should show a smaller coefficient of permeability than a thick one, since the passage of water through the filter skin is not affected by the clay below the skin. But the tests gave the same coefficient of permeability for a thin as for a thick layer. Again, after a test on a liquid-plastic sample, the surface was scraped off and the test was repeated, without any change in result.

In the moisture-permeability curves in Fig. 1, those points whose ordinates were determined by means of

the apparatus shown in Fig. 3 are indicated by circles. For moisture contents approaching the lower limit of the plastic state the coefficient of permeability became exceedingly small, so that it could no longer be reliably determined in a direct way. In order to overcome this difficulty and to make it possible to investigate the permeability for the semi-solid state of the clays also, I was obliged to work out an indirect method, which enables the permeability of almost impermeable samples to be measured comparatively rapidly. This method will be described in a subsequent article. It is distinguished by the fact that the water merely circulates in the interior of the sample, without entering or leaving it, since during such a process no filter skin can possibly

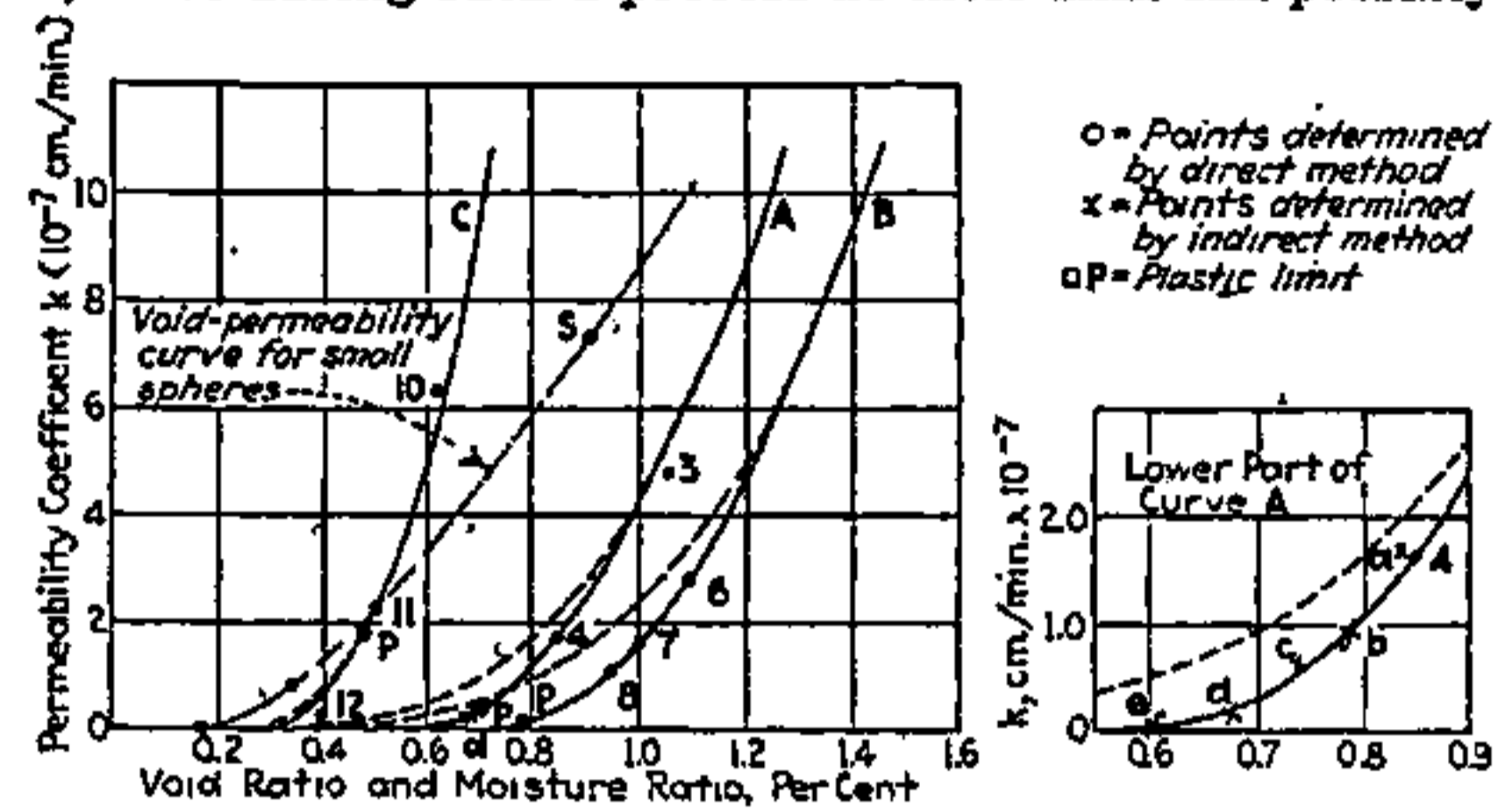


FIG. 1—PERMEABILITY-MOISTURE CURVES FOR THREE CLAYS

be formed. The close check between the points 'a' and 4, Curve A, Fig. 1, furnishes a conclusive additional proof for the results obtained by means of the apparatus Fig. 3 to be reliable.

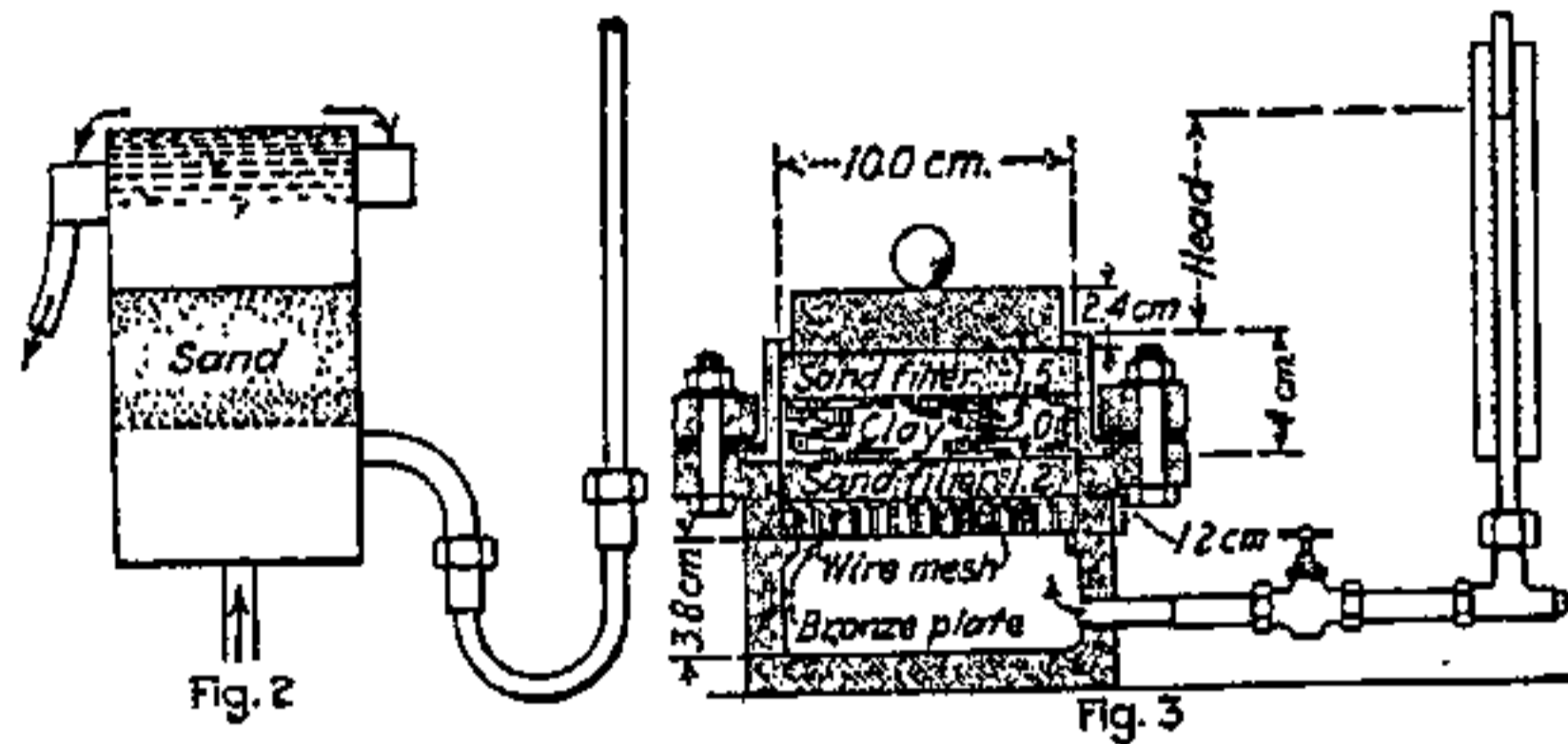
*Darcy's Law Valid*—Tests with the apparatus just described proved that Darcy's law holds for percolation through clay of plastic consistency. In addition it may be inferred from certain of the results that it is also valid for the flow of water through semi-solid clays. Distinct departure from Darcy's law could be noticed only for semi-liquid clays. Reducing the hydraulic grade from 50 down to 10 or 15 caused rapid decrease of the coefficient of permeability, and only at low heads was the coefficient fairly constant. This phenomenon, however, may be explained by the fact that the structure of a semi-liquid clay is honey-combed. The void-ratio of the loosest possible aggregation of equal spheres amounts to 0.91 (volume of voids 47.6 per cent), while for a semi-liquid mass of clay it is approximately 2. The structure of such a mass is stable merely because initial friction\* keeps the particles in relative position, and the average width of the voids is far greater than the average diameter of the clay particles. When water percolates through such a mass under a considerable head it produces elastic and non-elastic deformations and grain displacements similar to the deformations produced by a stream of water forced through a system of very expansible rubber tubes, but at lower hydrostatic pressure the elastic deformations disappear and the coefficient of permeability changes accordingly.

Hence we can state that Darcy's law is valid at least for the flow of water through clays with a medium or a low moisture content (plastic or semi-solid consist-

\*NOTE—The term initial friction has been used by the author for indicating the shearing strength the clay has, if it does not stand under the influence of any pressure (neither external nor capillary). It amounts to at least  $10^{-6}$  dynes per point of contact between each pair of clay particles.

ency). It remains to be examined whether or not the permeability coefficients agree with the formula given above.

*Departure from Theory*—Fig. 1 contains the results of three sets of permeability tests. In order to compare these results with the formula, I determined first such values of  $\frac{C}{v_0}$  and of  $d_w$  as would make the curve fit the test points 1 to 4 (curve A, points 1 and 2 outside of the figure). Then, with the same value of  $\frac{C}{v_0}$ , I determined two other values for  $d_w$ , to fit the respective groups 5 to 8 and 9 to 12 (curves B and C). If the



FIGS. 2 AND 3—MEASURING PERMEABILITY OF SAND AND CLAY

Fig. 2—Apparatus for sand. Fig. 3—Apparatus for clay.

formula is correct, the points of each set ought to be close to the corresponding theoretical curve. For medium moisture contents the curves and the points agree fairly well, but for moisture contents approaching the plastic limit the disagreement increases rapidly, and for the semi-solid state the coefficient is only a small fraction of the theoretical value. The plastic limit (limit between the plastic and the semi-solid state) is marked in the figure by the letter *P*. The anomaly becomes particularly obvious when comparing the relation between the void-ratio and the coefficient of permeability of sands with the results of the clay tests. Increasing the void-ratio of a sand from 0.6 to 0.8 increases the value of  $k$  by 75 per cent, whereas the same increase of the void-ratio of the clay increases the coefficient of permeability by several thousand per cent.

For reasons which have already been discussed, it seemed inadmissible to blame such important disagreement merely upon the approximate character of the assumptions. On the other hand there was reason for suspecting that the disagreement is due to a physical factor, one not present in flow of water through sand but appearing only when the voids are very small. If the disagreement were due to error of the formula, it ought to manifest itself whenever the void-ratio drops below some certain value but instead, it is invariably associated with the semi-solid state of a clay, regardless of whether this state (plastic limit) is reached at void-ratio 0.3 or 1.0. Experience has shown that the moisture content of a clay corresponding to the plastic limit increases with increasing fineness of the grains, provided the grains are of similar kind, so that it seemed likely that the plastic limit is reached as soon as the width of the voids drops below a definite critical value, regardless of what the effective size of the grains may be. If that be true, the observed anomaly concerning the permeability too should be in some way related

to the width of the voids dropping below a certain critical value.

*Change in Viscosity*—From this reasoning it seemed that the physical constants of the water might change when the capillary channels of a shrinking clay become reduced to a certain size. This supposition was strengthened by anomalous results obtained in certain evaporation tests.

Prisms made of different clays were allowed to dry at normal temperature (15 to 25 deg. C.), and their loss of weight was determined three times a day, together with the shrinkage of the prisms (distance between gage-points on their surfaces); at the same time the loss of weight of a cylindrical vessel containing distilled water with a wide, free surface was measured. Basing my calculations on the data thus obtained, I computed for each one of the prisms the ratio between the speed of evaporation of the capillary water and the corresponding speed of evaporation of the water contained in the vessel (relative speed of evaporation). The results of the computation were plotted in a diagram (Fig. 4) the abscissas of which represent water content (void-ratio) and ordinates represent the relative speed of evaporation. The letter *P* indicates the plastic limit, *N* the point at which the surface of the capillary water retires into the interior, and *S* the shrinkage limit. Down to the plastic limit the relative speed of evaporation was constant, at about 1.1. But between *P* and *N* the speed of evaporation decreased, though the surface of the capillary water remained at the surface of the prisms.

Thus, one of the physical constants, the relative speed of evaporation, changes after the state *P* has been reached. It has already been remarked that the surface tension of the water changes its value at this same point. Under these conditions I felt warranted in concluding that the physical constant which determines the speed of percolation, i.e., the viscosity of the water, also changes its value as the clay approaches the lower limit of the plastic state.

Deducing from the observed facts the law which connects width of voids and coefficient of viscosity, we obtained the law

$$v_0' = v_0 \left( 1 + \frac{R}{s^2} \right)$$

where  $v_0$  denotes the normal value of the coefficient of viscosity and  $v_0'$  the viscosity of the water flowing through a capillary channel  $2s$  (cm.) wide;  $R$  is a constant of value between the limits  $6 \times 10^{-10}$  and  $2 \times 10^{-8}$  (its exact value has not been determined, but knowledge of the limits is sufficient for our purposes). In voids smaller than  $0.1\mu$  (0.0001 mm.) the value of the coefficient of viscosity increases rapidly.

*Revised Permeability Formula*—Because of this change in viscosity the formula for permeability of clay had to be modified, the result being,

$$k = \left( \frac{C}{v_0} \right) \left( \frac{v_0}{v_0'} \right) \frac{(e - 0.15)^2 (1 + e)}{(e - 0.15)' + \frac{C}{d_w^2}} d_w^2$$

It is obviously difficult to determine the effective size  $d_w$  of the grains of clay because it requires wet mechanical analysis of the material. But as the formula expresses the relation between water content and permeability it can be used even when  $d_w$  is not known. The only requirement is that we know the result of a single permeability test on the sample of clay under

study. This one result, the value  $k$  of the sample at one particular moisture content, may be used to determine the value  $k$  at any other moisture content, the two values being connected by the law expressed by the formula given.

Thus suppose we have two clays, for one of which the effective grain size  $d_w$  is  $m$  times larger ( $m$  being unknown) than the grain size  $d_w$  of the other one. In addition, we suppose that the permeability of the clay with the grain size  $d_w$  be given by the formula

$$k_1 = 3.34 \times 10^{-7} \left( \frac{v_0}{v_t} \right) \frac{(e - 0.15)^2 (1 + e)}{(e - 0.15)^2 + 0.0166} \text{ (cm./min.)}$$

According to what precedes, the permeability of the clay with the grain size  $md_w$  should be

$$k_2 = 3.34 \times 10^{-7} \left( \frac{v_0}{v_t} \right) \frac{(e - 0.15)^2 (1 + e)}{(e - 0.15)^2 + \frac{0.0166}{m^2}} \text{ (cm./min.)}$$

By making a single permeability test with the second clay, one point of its moisture-permeability curve is obtained; and this point permits of solving the above equation for  $m$ . Knowing the value of  $m$ , the complete curve can be drawn from the formula and subsequent permeability tests should plot close to this curve. Curve B in Fig. 1 is an example of what agreement may be

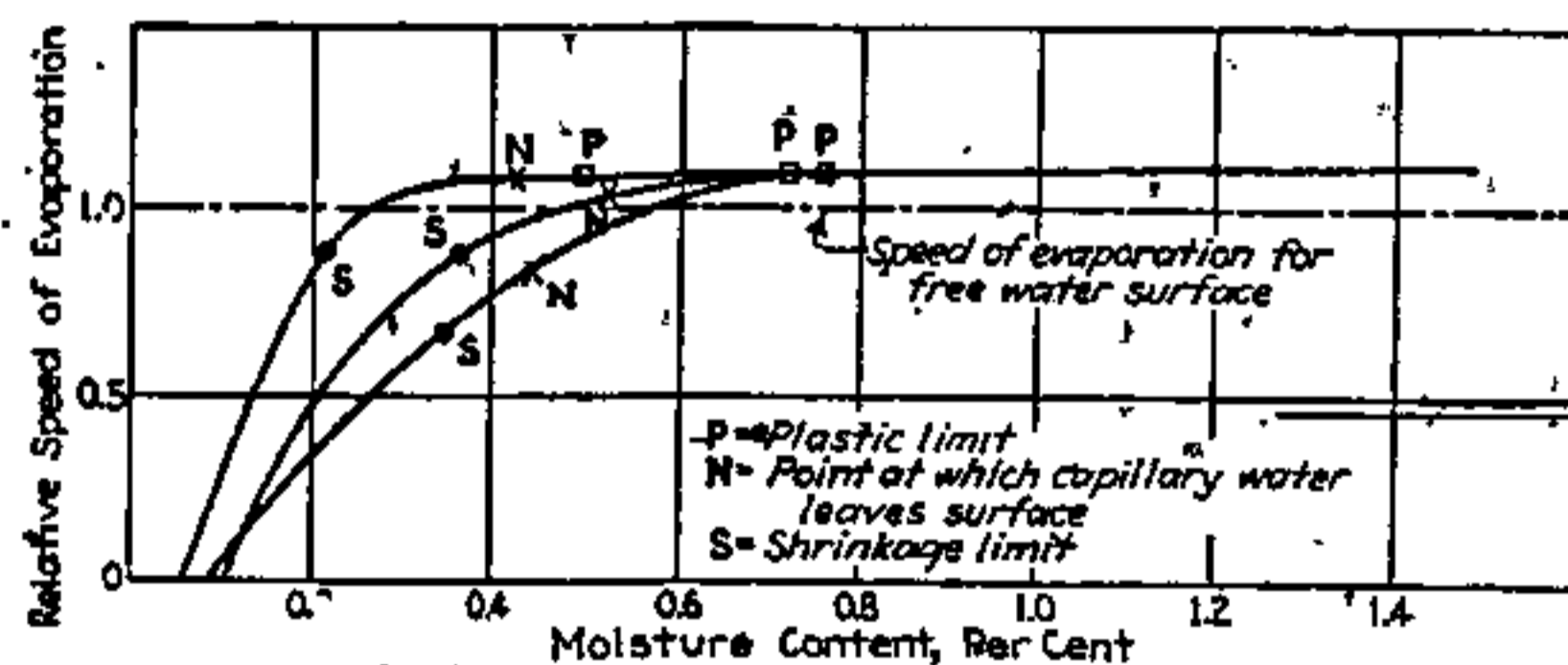


FIG. 1.—SPEED OF EVAPORATION OF CAPILLARY WATER OF CLAY

expected. For one point of this curve the test furnished the value  $k = 40 \times 10^{-7}$  cm./min., while the formula gave the value  $85 \times 10^{-7}$  cm./min. In the case of curve C, Fig. 1, the agreement between theory and test results was less satisfactory, on account of the high sand content of this material, amounting to 59.1 per cent. Such a soil may be called a sandy mud. Nevertheless its permeability also proved to vary between extraordinary wide limits as its density varies. This further illustrates that one cannot possibly describe the permeability of a clayey material except by a moisture permeability curve; a single figure means nothing.

*Individual Character of Different Clays*—For clays having equal grain size, the plastic limit may be high or low according to the shape of the grains and to the degree of adsorptive saturation. Therefore the values of the constants  $C$  and  $c$  in the general formula for the permeability coefficient depend on the nature of the grains as well as on their size, and an expression of the numerical type as just quoted is not valid except for clays whose grains are fairly similar in character. If the water which percolates through the clay contains substances in solution, these substances are adsorbed by the clay, and the solute accumulates along the surface of the clay particles. The concentration of the solution influences considerably the viscosity of the solvent, and in turn the viscosity of the solvent has a marked effect upon the permeability of the medium.

Attention should be called once more to the important

fact that the preceding discussions apply only to homogeneous mixtures of clay and water. Crumb clay soils do not belong to this class. Percolation through such soils inevitably leads first to the formation of a filter skin and then to the filling up of the widest voids with spongy colloidal matter, detached from the top layers of the crumbs. Hence percolation through crumb soils is associated with grain displacements, internal erosion and deposition. It is a very complex phenomenon, which still needs careful investigation. It is altogether inadmissible to simply apply to the hydraulics of crumb soils the conceptions derived from sand filter tests or those applicable to homogeneous clays.

<sup>1</sup>C. S. Slichter, Annual Report of U. S. Geological Survey, 1899, p. 311.

<sup>2</sup>"Filtergeometrie," by L. Darapsky, *Ztschr. für Mathematik u. Physik*, 1912, p. 170; "Geometrische Untersuchung der Bewegung des Grundwassers im Gerölle und der Wasserfiltration durch Sand," by L. Burmeister, *Ztschr. für angewandte Mathematik u. Mechanik*, 1924, p. 33.

<sup>3</sup>Erdbaumechanik, by K. Terzaghi (1925).

<sup>4</sup>Verbal Communication of R. R. Coghlan, chemist in charge; "Experiments on Material for Cold Springs Dam, Umatilla Project," by D. C. Henny and E. G. Hopson, *Eng. News*, 1907, p. 250; "Experiments on Cold Springs Dam and for a Projected Dam Which Was Abandoned," by T. A. Noble, *Eng. News*, 1907, p. 480.