Principles of Soil Mechanics:
IV—Settlement and Consolidation of Clay

Consolidation the Result of Decrease of Moisture Content Under Load—Change in Hydrodynamic Stress—Typical Computations—Application to Permeability Determination

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When a homogeneous layer of clay is loaded, the deformation of the clay increases for some time after the load is applied. This phenomenon is of great practical importance, being involved in all the numerous cases of settlement and subsidence met with in practice, as well as in the gradual consolidation of clap deposits.

The physical causes of this phenomenon have already been explained: The load produces a hydrostatic stress within the clay; which stress in turn causes the water in the clay to flow toward the surface, until stress equilibrium is reached, when the hydrostatic stress has decreased to zero. Basing calculations on this set of conditions, and on what has been developed in the preceding article on permeability, we can derive the relation between pressure, moisture content and time. That is to say, we obtain a solution of the time-settlement problem.

In Fig. 1 the pressure-moisture and moisture-permeability curves for a particular clay are given, as obtained by experiment. Suppose that a layer of this clay is enclosed in a rigid ring, and a load of intensity p_0 per unit of area is applied to its upper surface, which is covered with water and is protected by a sand filter. What change will the layer undergo if the pressure is suddenly increased to p, and then kept constant?

According to the diagram, the moisture equivalent of pressure p_0 is e_0, and the coefficient of permeability at moisture content e_0 is k_0. When the pressure is increased these quantities ought to change to the values e_1 and k_1. But as the increased compression involves the escape of capillary water it is a rather slow process, and in every stage of this process the distribution of the hydrodynamic stresses must provide for the hydraulic gradient required to drive the capillary water out. This fact represents the point of departure for the subsequent considerations.

The diagram Fig. 2 represents a cross-section through the layer; h is the thickness the layer would have if its volume of voids were reduced to zero (reduced thickness). The origin of co-ordinates is taken at the lower surface of the layer and abscissa zero represents pressure p_0. At the upper surface of the layer the pressure is constant and equal to P_1, and the hydrodynamic stress is zero. According to the physical facts as set forth in the curves of Fig. 1, no change of water content is possible without change of pressure in the solid skeleton of the clay. But as we observe that the water content of the layer decreases while the external load remains constant, we are obliged to conclude that the pressure acting within the solid skeleton gradually increases. Let dp be the increase of the pressure p per time element dt, at height z above the base of the layer. The sum of the pressure p, within the solid skeleton and the hydrostatic pressure w at the same point must be equal to p_1, or $p_1 = p + w$. Differentiating, $dp = -dw$.

To determine how the pressure changes with time, assume a straight-line relation between pressure change and the corresponding moisture or void-ratio change. This assumption is permissible so long as the total change from p_0 to p_1 is not large. The relation is expressed by the slope a of the pressure-moisture curve in Fig. 1. Then $de = -a\,dp = a\,dw$. The quantity of water which drains per unit of time through unit area of a horizontal cross-section at height z above the base is $Q = ks$, where k is the coefficient of permeability and s is the hydraulic gradient, equal to $-dw/dz$). Hence $Q = -k\,dw/dz$. But the hydraulic pressure w changes also with the time; i.e., while the water flows from elevation z to elevation $z + dz$, in the time element dt, the clay pressure p increases by dp, which equals $-dw$, and this change of the pressure involves a change of the water content according to the relation $de = -a\,dp$, given above. Hence the quantity Q increases on its way from elevation z to $z + dz$, and we have,

$$\frac{dQ}{dz} = -\frac{de}{dt} = a\frac{dp}{dt} = -a\frac{dw}{dt}$$

Inserting the value of Q from the previous expression we obtain,

$$-\frac{k}{a}\,\frac{d^2w}{dz^2} = \frac{dw}{dt}$$

This equation represents the distribution of hydrodynamic stresses in the clay. It is mathematically identical with the equation for linear flow of heat through a plate of isotropic material of thickness $2h$ and of uniform temperature, insulated on its lateral faces, which is transferred suddenly into a space of lower temperature. The rate at which cooling proceeds from upper and lower faces toward the interior corresponds to the progress of consolidation of the clay from its upper (loaded) surface downward into its interior. This thermodynamic analogy makes it possible to transform any time-settlement problem into a thermodynamic one, noting that heat content replaces moisture content e, specific heat replaces modulus of compression a, temperature replaces hydrodynamic stress w, and coefficient of heat conductivity replaces coefficient of permeability k. There are the further relationships that (1) in thermodynamics specific heat decreases with increasing heat content, and in the hydraulics of clays the modulus of compression decreases with increasing moisture content; (2) loss of heat causes a body to contract, and loss of water causes clay to shrink.

Similar analogies play a very important part in modern applied mechanics. Thus, there is a thermodynamic analogy for the flow of water towards a series of wells, and a hydrodynamic analogy for the torsional stresses in solid bodies. Another analogy is utilized in the soap film method of solving stress problems, recently proposed by A. A. Griffith. According to this method, one can determine the distribution of torsional stresses over the cross-section of a twisted bar by simply measuring the deflection of a soap film stretched across an opening whose outlines are identical with those of the section.

The partial differential equation covers the whole field of time-settlement problems for watersoaked clay soils, and the special character of the problem merely influences the mathematical method to be employed for solving the equation. In dealing with the problem of gradual consolidation of a layer of clay under uniform load, at lateral confinement, the equation was solved by means of Fourier's series. From the results, numerical values have been computed, based on the following assumptions: thickness of the layer, 20 m., reduced thickness of the layer = 13 m., initial pressure 4 kg./cm.^{2}, final pressure 5 kg./cm.^{2}, modulus of compression 0.00002 g^{-1} cm.^{5}, and coefficient of permeability 0.100 cm. year^{-1}. The values of the two latter constants (a and k) correspond to the properties of a fairly fat clay in a plastic state. The results of the computation are graphically represented in Fig. 3. The curves show how exceedingly slowly the compression proceeds from the surface of the layer toward the interior. This explains the gradual increase of the settlement of structures resting on the surface of strata of plastic clay. The dotted lines represent the results obtained by means of an approximate method, based on the assumption that the distribution of the stresses follows a straight-line law; they agree well with those of the accurate method.

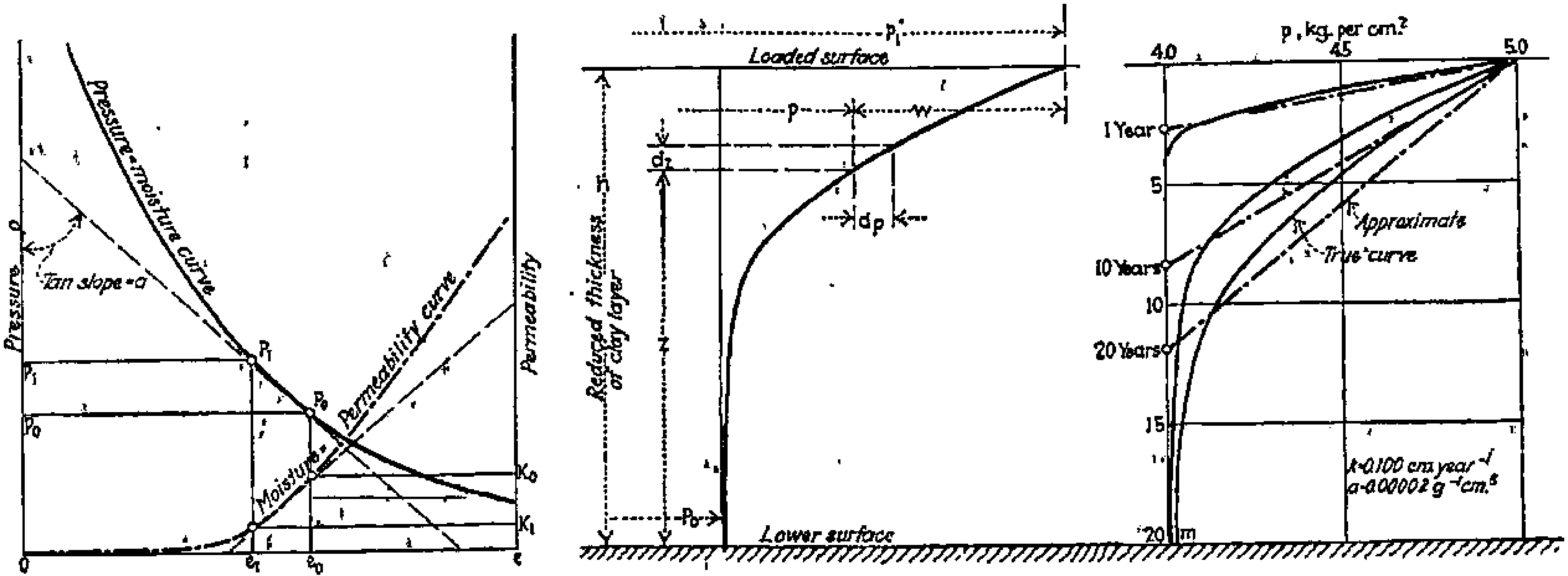

Fig. 1 Fig. 2 Fig. 3

FIGS. 1 TO 3—CONSOLIDATION OF A CLAY DEPOSIT UNDER LOAD

Fig. 1—Test curves for a clay sample. Fig. 2—Distribution of pressure through the layer. Fig. 3—Pressure distribution after different periods

SOME APPLICATIONS

A few simple examples may indicate what information the theory of hydrodynamic stresses is able to furnish.

Natural Settlement of Mud Deposits—Suppose a basin (lake or bay) is fed by a muddy stream so that mud is deposited at a uniform rate per unit of area and per unit of time. After a certain time the process of sedimentation stops. We inquire what will be the distribution of the stresses in the deposit at any definite time, and what will be the annual rate of settlement of the surface of the deposit.

We must be careful to distinguish between the hydrodynamic stress w acting in the capillary water, and the static pressure p acting in the mud. The ratio $w : p$ decreases, as time passes, but the sum of these two pressures constantly equals the external load, which, in our case, is represented by the weight of the mud located above the cross-section for which p and w should be determined. For a shrinking clay cube the external pressure is equal to zero, hence the hydrodynamic stress w is negative and the pressure p (in this case called the capillary pressure), is equal to it and of opposite sign. The internal friction is equal to the product of the static pressure p times the coefficient of internal friction, while the hydrodynamic stress w cannot produce any internal friction. Hence a decrease of w at a constant external load produces not only an increase of p but a corresponding increase of the internal friction. These fundamental facts ought to become as familiar to the foundation engineer as Hooke's law to the structural engineer.

In Fig. 4, diagrams b and c represent graphically the results of the calculation for a mud deposit formed at the rate of 5 cu.cm. of solid matter per square centimeter of bottom area per year; b corresponds to a sandy mud and c to a mud with a high percentage of colloids. The dotted lines passing through the upper corners of the graphs b and c represent the stress distribution for hydrostatic equilibrium, reached after infinite time. The full lines divide the abscissas of the dotted lines into two unequal parts, the right-hand part corresponding to the hydrodynamic stress and the left-hand part to the static pressure. At the lower end of each full line (pressure-distribution line) is written the time which elapsed between beginning of sedimentation and state represented by pressure distribution.

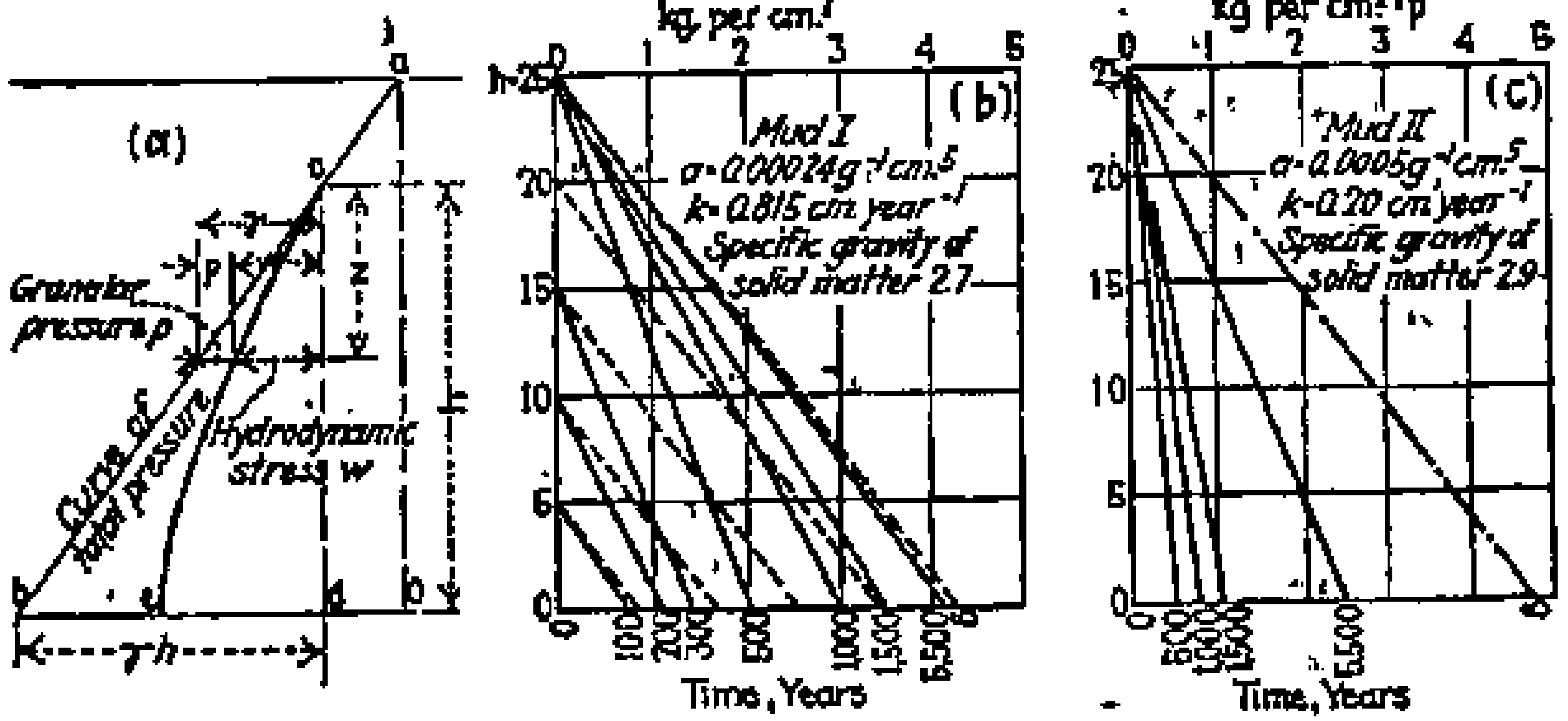

FIG. 4—HYDRODYNAMIC STRESS AS RELATED TO AGE OF DEPOSIT

From the diagram *c* in Fig. 4 (mud rich in colloids) it may be seen that 5,000 years after the deposit was formed the hydrodynamic stresses still amount to about one-half of the pressure exerted by the dead weight of the deposit. This fact indicates in turn that the internal friction of the mud amounts to only one quarter of what it will be after the state of hydraulic equilibrium has been reached. As the deposit represented by *b* is more permeable and less compressible than the deposit *c*, it is at that time (5,000 years after it was formed) almost in hydrostatic equilibrium.

A short time ago I had an opportunity to examine fresh samples extracted out of a young and very colloidal mud deposit by means of test borings. The tests clearly evidenced that the pressure corresponding to both the moisture content and the internal friction of the mud amounted to a small fraction only of what would correspond to the state of hydrostatic equilibrium. On account of the excessive hydrodynamic stresses, the bearing capacity of the deposit was so small that the plan of loading its surface with factory buildings had to be abandoned. After a few thousand years more, the same deposit would undoubtedly present a fairly good foundation.

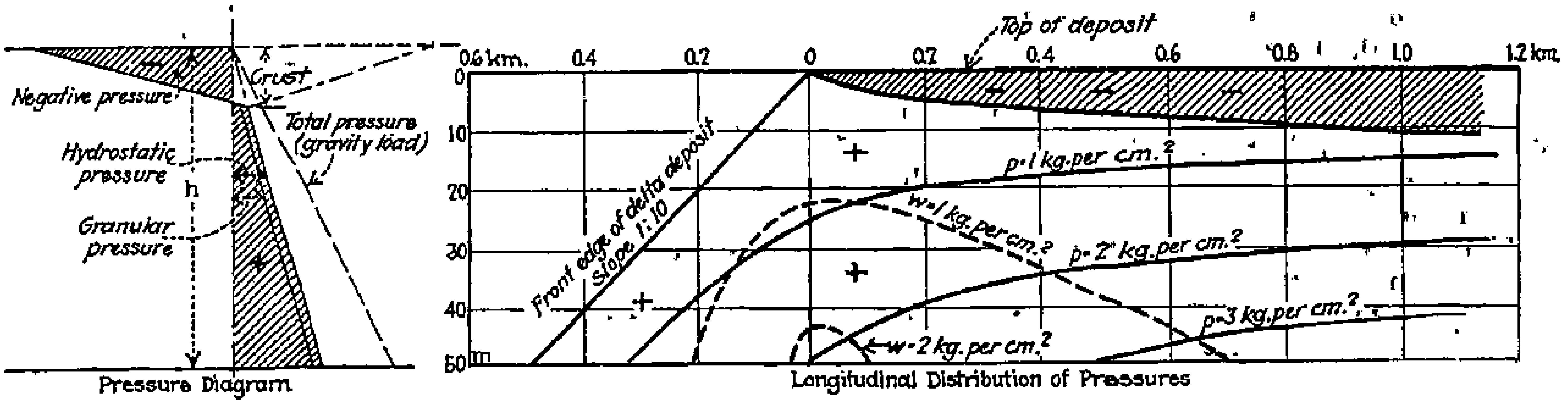

FIG. 5—STRESS DISTRIBUTION IN A DELTA DEPOSIT
Drainage by gravity and by evaporation. Negative hydro-dynamic pressures indicated by minus signs.

Impervious and Porous Layers—Suppose, now, that the deposit just discussed, instead of being homogeneous, included a mud layer less permeable than the remainder of the material. Such a layer has precisely the same effect on the drainage process as a layer of poor heat conductor would have on the cooling of a deep stratum of hot substance. Above the layer the mud will drain more rapidly, while below the layer it will remain almost liquid for a long time, and there will be a considerable difference of hydrodynamic stress between the two sides of the intermediate layer. If, on the other hand, the deposit contained a sand layer instead of the colloidal layer, the hydrodynamic stresses above and below the layer would be equal or nearly so. A hole drilled down to the sand layer would furnish artesian water, and drainage would proceed more rapidly than before, with correspondingly accelerated settlement.

On two occasions I had an opportunity to observe such a phenomenon in practice. In the first case the mud deposit had a thickness of about 15 m. Immediately after the sand stratum was opened up by means of a caisson well, some buildings located in the vicinity of the well started to settle badly. During the first year following the construction of the well, the settlements amounted to several inches and their rate of increase decreased from year to year, as theory demands. The well furnished artesian water. In the other case the mud deposit had a thickness of 50 m. While drilling a test hole through the mud we struck a layer of sand at a depth of about 15 m. To the surprise of the drill men, we got a rush of artesian water into the casing, although no artesian conditions were known to exist in the vicinity of the site.

Evaporation from Mud Deposits—Deposits of soft mud whose surface is exposed to the air gradually dry out. Experience shows that this drainage by evaporation leads to the formation of a semi-solid or solid crust, whose thickness increases as time goes on. I have previously proved that the solidification of a shrinking clay is caused by the capillary pressure, which in turn is associated with a negative hydrodynamic stress of equal intensity.

Consideration of the physics of the evaporation process leads to the conclusion that during the drying process two successive stages must be distinguished, the first stage lasting from the beginning of evaporation to the time when the capillary pressure at the upper surface of the deposit reaches the value of the transition pressure. At equal speed of evaporation and equal value of transition pressure, the thickness of the crust increases during the first stage in direct proportion to the permeability of the material. If the materials of two deposits are identical, the thickness of the crust at the outset of the second stage will be in inverse proportion to the speed of evaporation. During the second stage the rate of increase of the thickness of the crust becomes exceedingly small and, in opposition to what is true for the first stage, almost independent of the temperature of the atmosphere.

Basing my computations on the results graphically represented in diagram *b* of Fig. 4, and on the deductions just stated, I surveyed the stress distribution existing in a delta deposit whose outer edge advances toward the ocean at the rate of 1 m. per year and whose capillary water evaporates along the horizontal top surface of the deposit. The outcome of the investigation is represented graphically in Fig. 5. The hydrostatic pressure within the capillary water of the crust is negative, that below the crust positive. The moisture content of the deposit just below the crust must be a maximum. If a canal is dug in the deposit to a greater depth its bottom will discharge water, while those parts of the slopes which are located within the crust will absorb water.

The positive hydrostatic pressure in the capillary water of the core of the mud deposit is the greater, the more rapidly the deposit was formed and the less permeable the deposit is. Due to the assistance of this excess pressure, natural deposits of highly colloidal mud are apt to flow out spontaneously, apparently without any external cause. Such phenomena have been observed and they are known as "submarine landslides."

Conditions similar to those represented in Fig. 5 exist in hydraulic fill dams, for some time after they have been deposited. The less permeable the core material, the more important are the hydrostatic pressures in the capillary water of the core and the more imminent is the danger of a dam failure by lateral eruption of the core material.

Indirect Determination of Permeability of Clays—The theory of hydrodynamic stresses led to a very simple method for determining the coefficient of permeability of plastic or semi-solid clays. Suppose a layer of clay enclosed within a rigid ring and its surface covered with water is in hydrostatic equilibrium. If a surcharge is applied to its upper surface, there results in the capillary water a positive hydrostatic pressure whose intensity increases from the upper surface towards the bottom, while the simultaneous static pressure within the solid skeleton of the mass decreases from the top towards the bottom. If the load is kept constant, water will slowly be squeezed out. But if, instead of keeping the load constant, we keep the compression constant, by fixing the head of the testing machine in its new position, the moisture content of the layer can not change. Immediately after the head of the testing machine has assumed its new position, the hydrostatic pressure will be a maximuum near the bottom and a minimum near the top surface of the sample. As a consequence the water will move in an upward direction, causing the top part to swell, and the pressure acting against the head of the machine will decrease. Since the speed of the decrease of the pressure depends on the coefficient of permeability of the clay, the value of k can be calculated from rate of decrease of pressure.

Tests involving this action were made with an apparatus similar to that used for making load tests on saturated clay in confined condition. It consists of a zinc vessel, two bronze rings and a sand filter. The clay sample had a diameter of 8 cm. and a thickness of 4 cm. Pressure was applied by a screw testing machine. Fig. 6 gives the results. First a pressure of 14.1 kg./cm.² was applied and then the head of the machine was held fixed. During the following hours the pressure decreased and gradually approached the value 1.72 kg./cm.² (curve I). At the same time the rate of decrease of the pressure became slower, approaching zero. Let h be the reduced thickness of the layer, e_1 the moisture content, a_1 the modulus of compression (derived from the main branch of the pressure-moisture curve), a_2 the modulus of expansion (derived from the resaturation line of the pressure-moisture diagram), and k_1 the coefficient of permeability, all taken at the pressure p_1, and c, a coefficient whose value can be calculated from the speed with which the pressure acting against the head of the testing machine decreases; then

$$k_1 = c_1 \frac{a_1 h^2 (1 + e_1)}{2\left(1 + \sqrt{\frac{a_1}{a_2}}\right)}.$$

Two days after the test was started, the pressure was raised from 1.72 kg./cm.² to 27.3 kg./cm.², and the machine again held fixed. For this second stage of the test, the relation between pressure and time is represented by curve II. Similarly curves III and IV were derived, for still higher pressures.

By means of these tests it was possible to investigate with accuracy the flow of water through clays compressed by a pressure so high that their coefficient of permeability amounted to not more than 0.000,000,007 cm./min. Hence the sensibility of the method is practically unlimited.

The results of these tests proved conclusively that the flow of water through clays even of semi-solid consistency follows Darcy's law as closely as does the flow of water through sands.

Value of the Theory—Suppose an engineer observes that a structure founded on a bed of clay has suffered important settlements, and that the settlements increase in course of time at a definite rate; he publishes his observations and adds according to traditional practice, some data concerning color and moisture content of the clay. A few years later another engineer wants to find out whether a clay stratum at some other place will behave under load in a similar way. He will discover that he cannot. Or, if he is innocent enough to believe he can, with no other basis for conclusions than the data published by his predecessor, he will probably have later opportunity to revise his optimistic ideas.

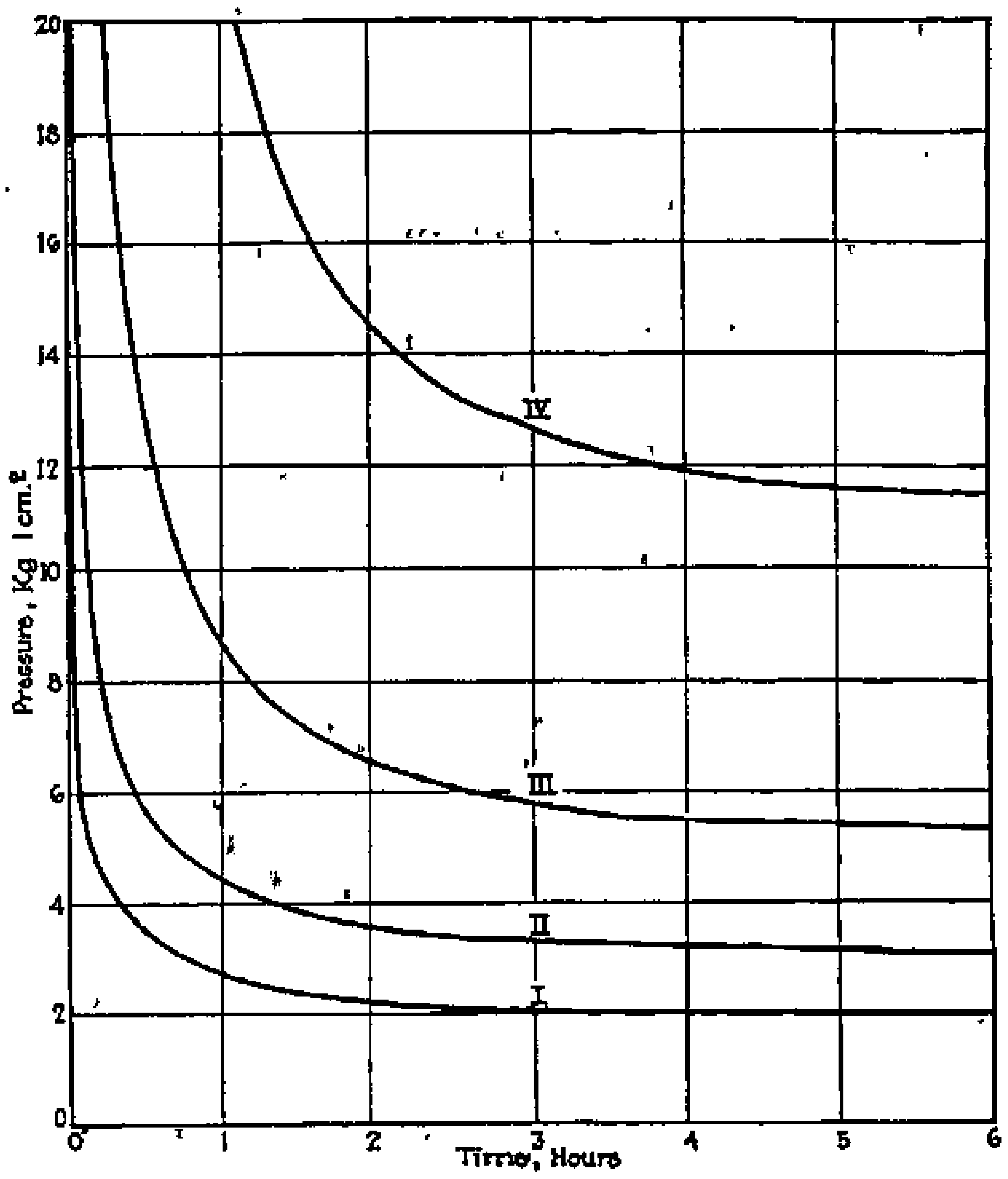

FIG. 6—PRESSURE-TIME CURVES FOR LOADED CLAY LAYER

At constant moisture content. The four curves represent different initial applied pressures. In each of the four tests the deformation was maintained constant after the load had been applied.

But with the theory of hydrodynamic stresses at our disposal to explain the time settlement phenomena on a physical basis we are better equipped. The fundamental differential equation does not involve color or moisture content of the clay, but reveals the fact that the quantitative side of the time-settlement process depends only on the ratio a/k; everything else is nonessential for this particular process. Two clays may behave altogether differently in spite of their having the same blue color and the same moisture content, while two other clays of different color and different moisture contents may behave identically provided their a/k ratios are identical.

In order to compare two clays as to settlement, we must submit a sample of each to a test of such kind that the outcome of the test depends on nothing but the value of a/k of the clay. Several test arrangements are possible which satisfy this condition and, as a matter of course, we will adopt that one for which our laboratory equipment is best fitted. By comparing the results of the tests made with the two clays, we are in a position to foretell with a fair degree of accuracy how similarly or how differently the two clays will behave as to the relation between time and settlement.

The time-settlement relation, however, is only one of the many facts which may interest the earthwork-engineer. If he wants to investigate the tendency of clays to slide or the quality of core materials for hydraulic-fill dams he must execute other tests. These other tests must be planned according to the physical theory of earth slides and of the stability of dams, and with appropriate consideration of the constants which are essential to the processes in question.

No theory is required for *making* the tests. But on the other hand it is not probable that the tests will be properly planned unless the theory of the action is understood. That is the reason why the author felt obliged to start with the causative relations between the elementary facts of soil mechanics and the general laws of physics. However, the mental inertia which besets work in this field seems to be enormous, and it will still be a long time before engineers generally will see more clearly in that respect. Though more and more time and money are being spent in experimenting in the field of soil mechanics, few experimenters yet realize the fundamental difference between basing the analysis of test results on vague traditional concepts such as "internal friction" of sands or "cohesion" of clays, on the one hand, and on the other hand rigorously tracing the observed phenomena back to their physical sources—pure surface friction, structural factors, surface tension of the capillary water, and hydrodynamic stress.