Principles of Soil Mechanics:
VI—Elastic Behavior of Sand and Clay

Testing Sand for Compressibility and Elasticity—Expansion and Resaturation—The Ideal Sand Cube and the Effect of Lateral Expansion—Comparison with Solids

By Dr. Charles Terzaghi

Professor of Civil Engineering, American Robert College, Constantinople, Turkey
Temporary Lecturer, Massachusetts Institute of Technology, Cambridge

IN STUDYING the difference in compressibility of sand and clay, which constitutes the last item of the observed characteristics as listed in the preceding article (*Engineering News-Record,* Dec. 3, 1925, p. 912), the elastic properties of sand were investigated. For this purpose a method was used similar to that previously applied to tests of the elastic properties of clay. The sand was poured into a steel ring resting on top of a polished cast-iron slab, and either left as deposited or compacted by hammering the outside of the ring. The surface was leveled with the ring, covered with a circular cast-iron plate, and loaded either by the lever of a home-made loading device (low-pressure tests) or by the piston of a screw testing machine (medium- and high-pressure tests). The diameter of the ring used for the medium- and high-pressure tests was 15 cm. and its height 4 cm. Compressions were measured by micrometer with the help of the interference contact indicator. Tests on dry sand and on sand completely immersed in water showed no difference.

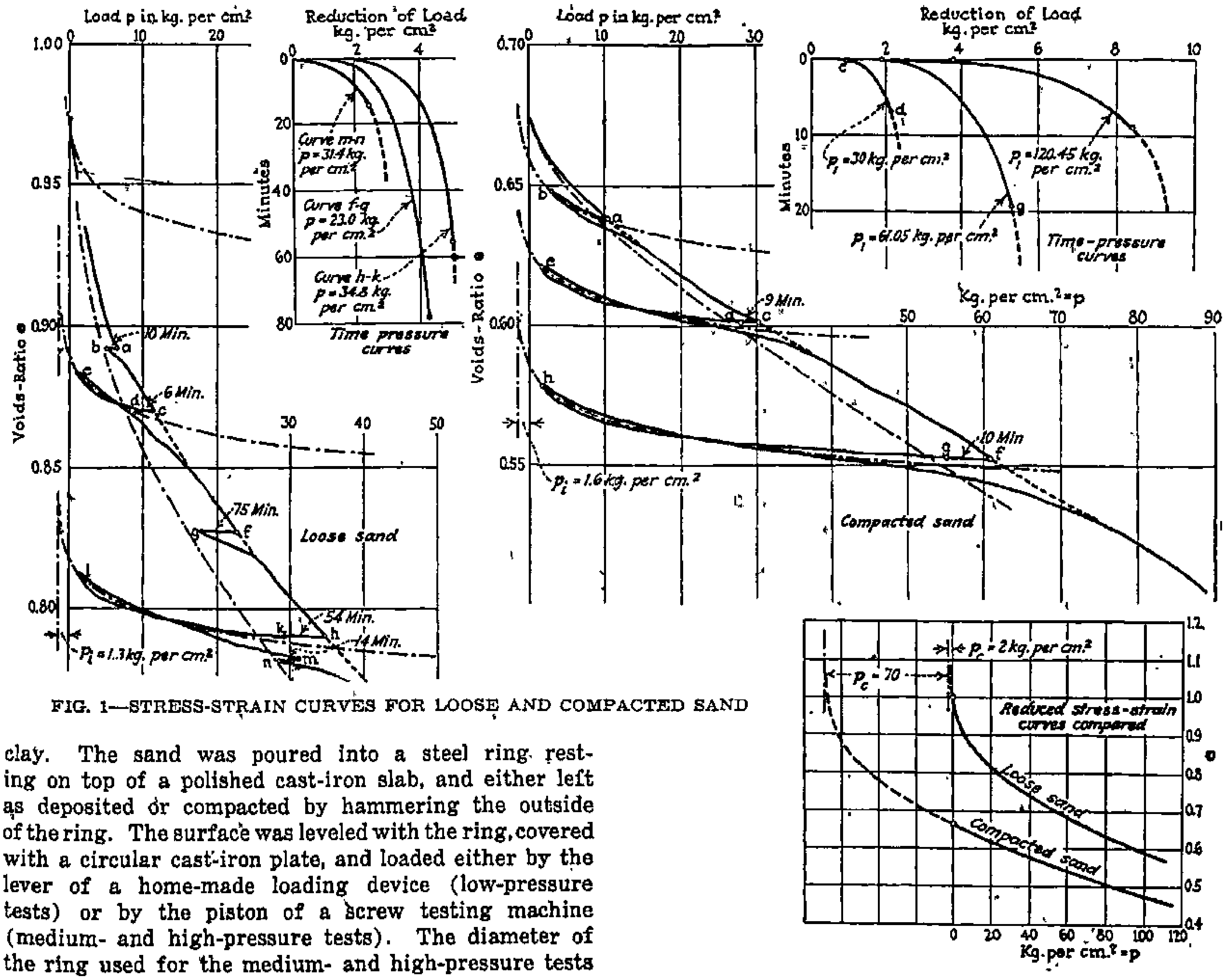

FIG. 1—STRESS-STRAIN CURVES FOR LOOSE AND COMPACTED SAND

Fig. 1 gives in separate diagrams the results of tests on loose and thoroughly compacted sand. In both cases the sand was a pure quartz sand produced by crushing white quartz pebbles, the product being sifted and washed; the grain diameter was 0.25 to 1.00 mm.

In the test on loose sand the initial volume of voids was 49.75 per cent (void-ratio 0.99). By means of a screw testing machine, pressure was increased at the rate of about 1 kg./cm². per minute to about 7 kg./cm². Then the machine was stopped for 10 min. (horizontal line *ab*) during which time the pressure exerted by the sand against the stationary piston decreased to about 5.5 kg./cm². Thereafter the machine was again started (curve *bc*), the load increasing to about 12 kg./cm². Again the machine was stopped, for 6 min. (*cd*), and then the load was released down to about 1.5 kg./cm². and reapplied (*ef*), forming a complete hysteresis loop and continuing the original stress-strain curve. A second load cycle was tried at higher load (loop *klm*).

The time-pressure diagrams, (at right and above in Fig. 1) show the pressure-decrease while the machine was stopped. These curves are very similar to the time-pressure curves for clay cubes, and in fact they have the same simple differential equation. The gradual decrease of the pressure is due to a gradually proceeding mutual compensation of unbalanced frictional resistances which at first take up part of the load.

If, instead of keeping the compression constant, we keep the load constant, the compression increases at decreasing rate, and the relation between time and compression is in every respect similar to the relation between time and compression for clay cubes under constant load.

Expansion of Sand—In the test on compacted sand, Fig. 1, the initial volume of voids was 40.2 per cent (void-ratio 0.673). The stress-strain curve is less steep than that for loose sand, while the hysteresis loops of both diagrams are almost identical. The close relationship between these diagrams and the corresponding diagrams for clay is obvious. In both, the loading cycles form curved hysteresis loops. In the sand diagrams, Fig. 1, the recurrent branches of these loops are expansion lines, but they correspond in every respect to the resaturation lines of the clay diagrams, and like the latter they are logarithmic lines, $e = A \log (p + p_i) + C$, where p is the external load (kg./cm.²), p_i is an initial constant having the dimensions of a pressure, A is the *expansion coefficient* of the sand, and C is a constant depending on the initial density of the sand. The expansion coefficient of sand corresponds to the resaturation coefficient of clay.

The expansion coefficient is almost independent of the density of structure of the sand. It was found to be equal to 1/100 for sand with very smooth grains and 1/176 for a sand with very rough grains. A very fat clay showed a resaturation coefficient of 1/22.3, a leaner clay 1/52.7, and a sandy mud 1/73. Thus sand is far less elastic than clay. The more sand a clayey soil contains the nearer its resaturation coefficient approaches the expansion coefficient of clean sand.

On the other hand the "initial pressure" p_i depends not only on the original density of the sand but also on the pressure at which expansion begins. This effect is due in part to the sand grains wedging together, in part to breaking of grains by the pressure, which in turn affects the structure. As illustrating the latter action it may be mentioned that a sand which initially was clean and dust-free was found, after its surface had been loaded to 50 kg./cm.², to contain 4.6 per cent of dust particles. The average value of p_i for sand (1.5 kg./cm.² for medium to high pressures) is very large, compared to the value for clay (approximately 0.002 kg./cm.²).

The effect of the unbalanced internal stresses on the stress-strain curve depends on the rate of loading. Under indefinitely slow application of load we would obtain the *reduced* stress-strain curve, shown by a dash line. How this line depends on the original density of the sand is represented by the third diagram in Fig. 1.

Fig. 2 shows typical stress-strain diagrams for a fat clay, a loose sand and a dense sand. The figure shows plainly that the difference between the elastic properties represented by the three curves is one of degree only. This difference, item (6) of the differences listed in the preceding article, Dec. 3, is a self-evident consequence of the fact that clay particles are scale-like. Sand can be compared to a pile of broken stone, while clay resembles a mass of flakes of paper. The higher the sand content of a clay, the less compressible and the less elastic it is.

Poisson's ratio for clay was computed from the lateral pressure in a loaded layer whose lateral expansion was prevented by a rigid enclosing ring. In a similar way Poisson's ratio for sand was determined. It has the value of 5.0 (as compared with 11 for Cooper sandstone, 5.1 for Troy granite, and 4.5 for Tuckahoe marble). Thus, while Poisson's ratio for clay is approximately identical with that of metals, its value for sand corresponds to the average Poisson's ratio for crystalline rocks.

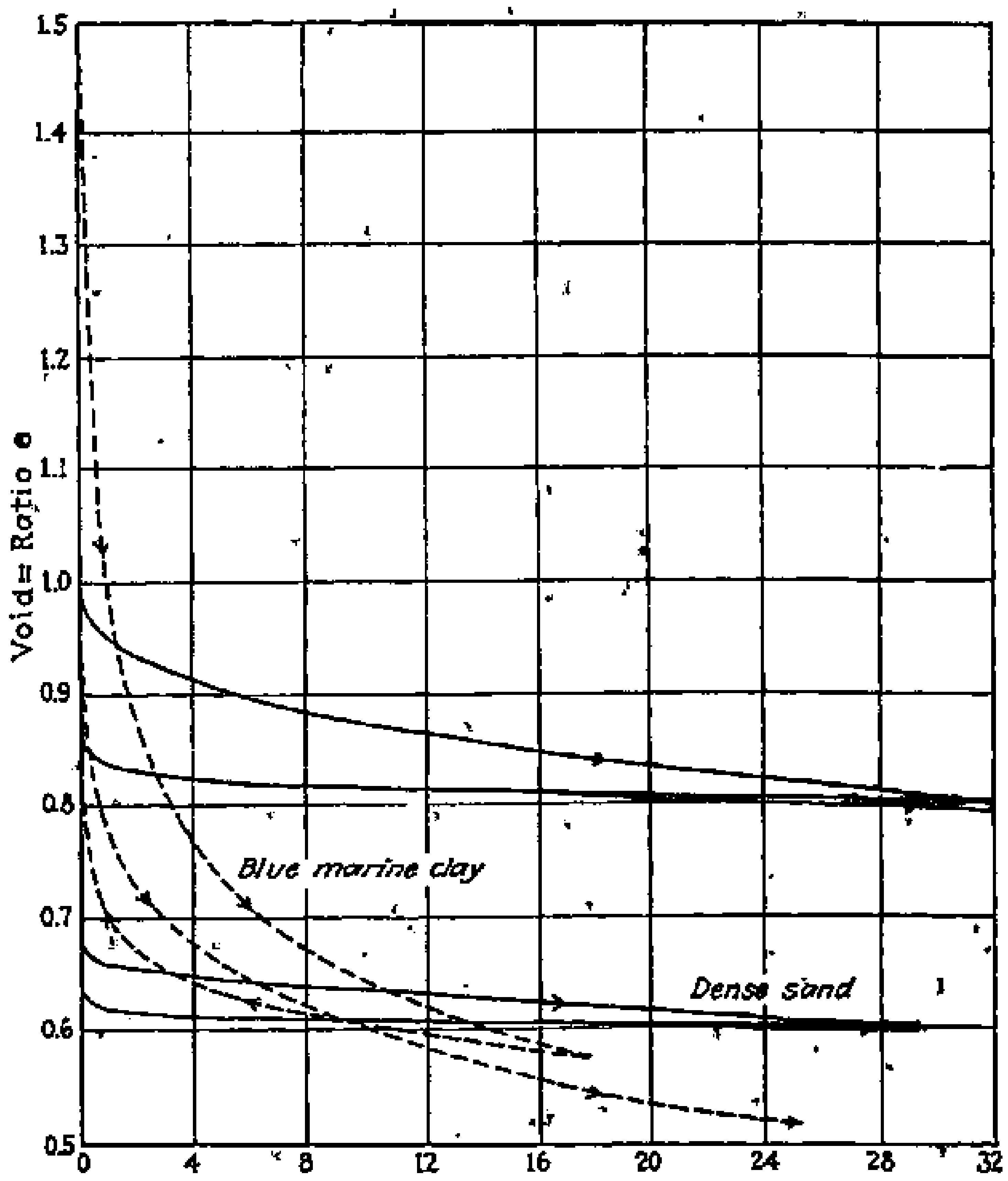

FIG. 2—PRESSURE-VOID (STRESS-STRAIN) CURVES FOR CLAY AND SAND

Modulus of Elasticity of Sands—Clay is capable of being tested in the form of an unconfined cube. The particles of such a cube, before loading, are subject to no force except capillary pressure, acting like a hydrostatic pressure. Suppose the clay particles are replaced by sand particles without changing the intensity of the internal pressure; it is readily inferred that, as sand and clay have quite similar stress-strain diagrams for compression when confined laterally, the stress-strain curve of the sand cube would be of the same type as that of a clay cube, which in turn was found to resemble closely the stress-strain diagrams for a concrete cube.

Of course it is not possible to test sand cubes directly. On account of the large voids of sand the capillary pressure is very small, too small to hold the particles together. But on the other hand each cube-shaped element of the backfilling of a retaining wall is comparable to our ideal sand cube; capillary pressure is replaced by the confining pressure of the surrounding material. When the retaining wall is forced horizontally against the backfilling, a cubical element of the backfilling is subjected to forces quite like those acting on a compressed clay cube. Hence the stress-strain diagram ought to be of similar form. Experiment fully confirms this.

When a retaining wall yields under the pressure of the backfilling, any cubical element of the backfilling undergoes horizontal elongation. This corresponds precisely to the strain effect of a pull exerted on the ends of a clay prism. In such a clay prism the principal stresses at right angles to the pull are constant and equal to the capillary pressure, while the principal stress acting in the direction of the pull is a variable compressive stress, equal to the difference between the capillary pressure and the pull. This residual stress must obviously be a compression, not a tension, because the pull cannot possibly exceed the intensity of the capillary pressure. But this residual compressive stress decreases as the elongation of the prism increases. In the same way, the lateral earth pressure exerted by a cohesionless mass of sand against a yielding retaining wall must evidently decrease as the yield of the wall increases.

A confirmatory experiment is represented in Fig. 3. Two rigid vessels A and B, of similar form, are filled with sand as shown. Close to the side of each vessel and just above midheight there is within the sand a horizontal steel tape set on edge, between two sheets of smooth paper. In vessel A the tape is pressed against the wall by what I have called the *earth pressure at rest* of the sand, since near the tape the sand undergoes no lateral expansion after its deposition. In vessel B, on the other hand, the upper half of the sand body rests on the surface of the compressible lower half of the sand body, and this lower half compresses during the filling process, each layer moving downward and (because of the splay of the sides of the vessel) expanding laterally as it moves down. According to the theory above suggested this expansion ought to decrease the lateral pressure of the sand. Individual measurements of the force required to pull the tape are plotted in the figure separately for vessels A and B; in each case the lower group of points refers to the unloaded sand, while the upper set was obtained when a weight of 25 kg. rested on the sand surface. The figure shows that the lateral earth pressure was 43 per cent smaller in vessel B than in vessel A—obviously due to the lateral expansion of the sand.

A different effect resulted when the sand was subjected to a violent disturbance. The bottom of vessel B was placed on removable supports and, after a first series of pulling tests was made, this bottom was lowered suddenly a distance of 2 mm. Immediately thereafter the lateral pressure was somewhat smaller than before, but in time it increased, went beyond its original value, and after 24 hr. became as great as the lateral pressure in vessel A. The increase of the pressure was obviously due to the gradual mutual compensation of internal frictional resistances similar to those which cause the pressure in a clay cube or a layer of sand to increase when its loading is decreased and the height of the cube or layer is then held constant.

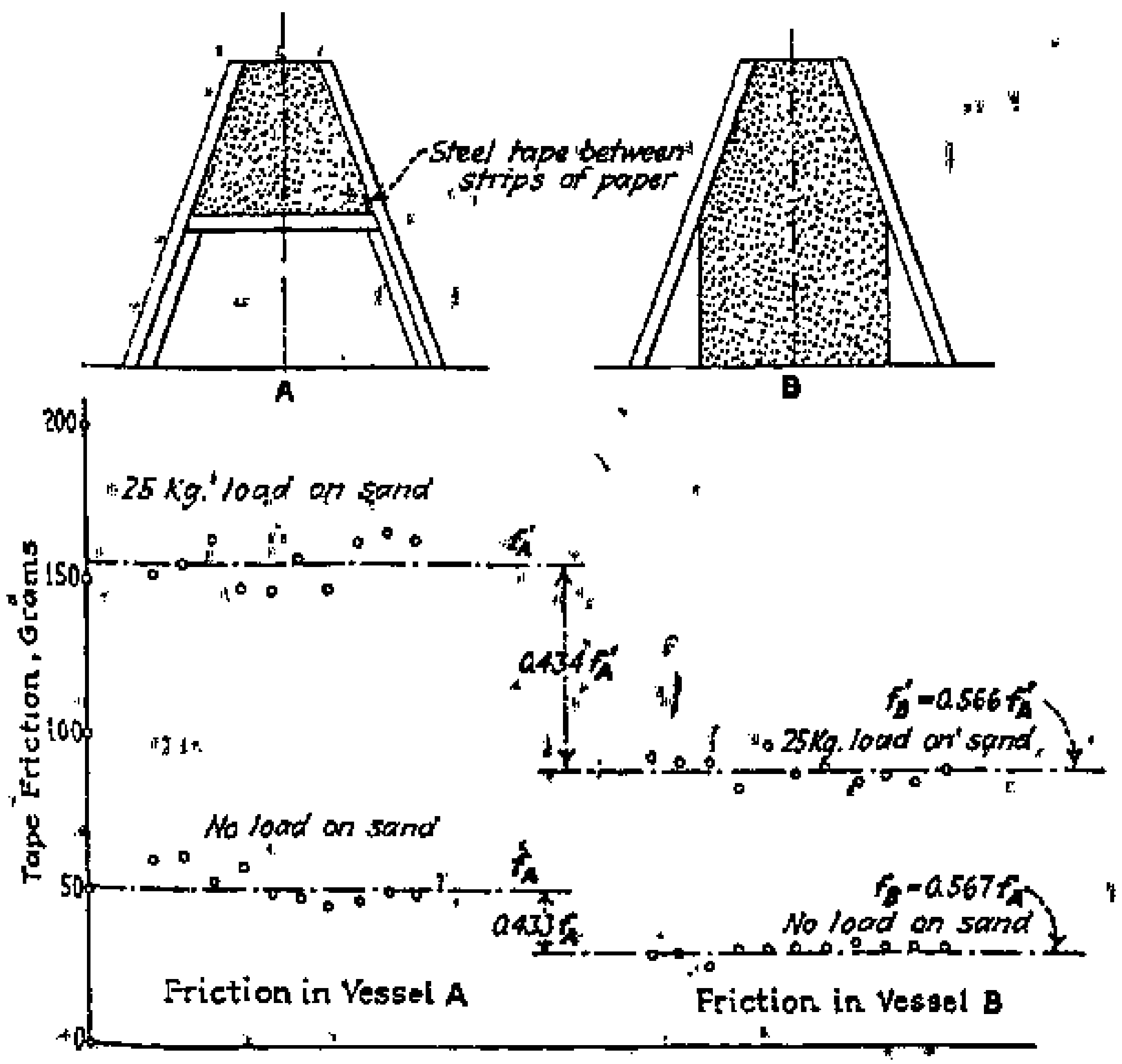

FIG. 3—LATERAL PRESSURE OF SAND (Influence of yield and settlement).

Further evidence for the pressure-relieving effect of lateral expansion of a mass of cohesionless sand has been furnished by various earth-pressure tests of G. H. Darwin, A. D. Donath, and the author.

Some interesting relations may be deduced from the stress-strain curves of sand drawn in Fig. 4. Compressing a mass of sand held in a rigid ring gives a stress-strain curve like the full line. The lateral pressure is smaller than the vertical pressure (load pressure) within the sand, and corresponds to earth pressure at rest. From the full-line curve can be derived (by the use of Poisson's ratio) the curve of cubical compression, representing the strain-effect of pressure equal in all directions (and therefore comparable to a capillary pressure); this is shown by a dash line. Suppose, now, the compression process represented by this dash curve be stopped at pressure p_1. Each element of the mass is now in the same condition as a clay cube subject to capillary pressure of the same amount p_1. Hence, if the sand specimen is further loaded in the vertical direction only, holding the lateral pressure constant at amount p_1 and allowing free lateral expansion to take place under the action of increased vertical load, we are bound to obtain a stress-strain curve identical with that of a vertically loaded clay cube. Two such curves are shown in Fig. 4, beginning at the respective cubical pressures p_1 and p_2; as the tests on clay cubes showed that the ratio between modulus of elasticity and internal pressure is constant, we may conclude that the slope of the hysteresis loops in these two curves will have the ratio $p_2 : p_1$. In other words, all stress-strain curves of the type shown for the two free cubes in Fig. 4 will differ only in the scale of their abscissas; so that if curve p_1 was obtained in a test, the curve p_2 can be drawn by simply increasing the abscissas of p_1 in the proportion $p_2 : p_1$. The truth of this statement was repeatedly checked by experiments on clay.

Clay and Sand Compared With Solids—In discussing the cohesion of clays I have previously shown that the ratio between the modulus of elasticity and the internal (intrinsic or molecular) pressure is a constant not only for clay but also for metals. Therefore *the laws expressed by Fig. 4 are approximately valid for the whole field of materials*, regardless of whether the particles of the material are kept together by molecular attraction (solids), by a capillary pressure (clays), or by the pressure due to the weight of the material itself (cohesionless sands).

Let us imagine for instance a cubical body consisting of cohesionless sand. From the tests already made we may calculate that an internal pressure of 1226 kg./cm.^{2} will bring the modulus of elasticity up to 200,000 kg./cm.^{2} This value is approximately equal to the value of the modulus of elasticity of ordinary concrete. The compressive strength of the concrete, however, amounts to not more than 300 kg./cm.^{2}, while the cube of sand can carry a maximum load of about 10,000 kg./cm.^{2}, provided the internal pressure remains unchanged. For loads ranging between zero and 300 kg./cm.^{2} the relations between stress and strain for sand agree far better with Hooke's law than do those for concrete. This statement may serve as one of the many examples of the fact that in solid bodies the ratio between compressive strength and intrinsic pressure is exceedingly small, compared to the corresponding ratio for aggregates of individual grains. Investigation of the physical causes of the low value of this ratio for solids represents one of the most attractive problems of modern research on resistance of materials.

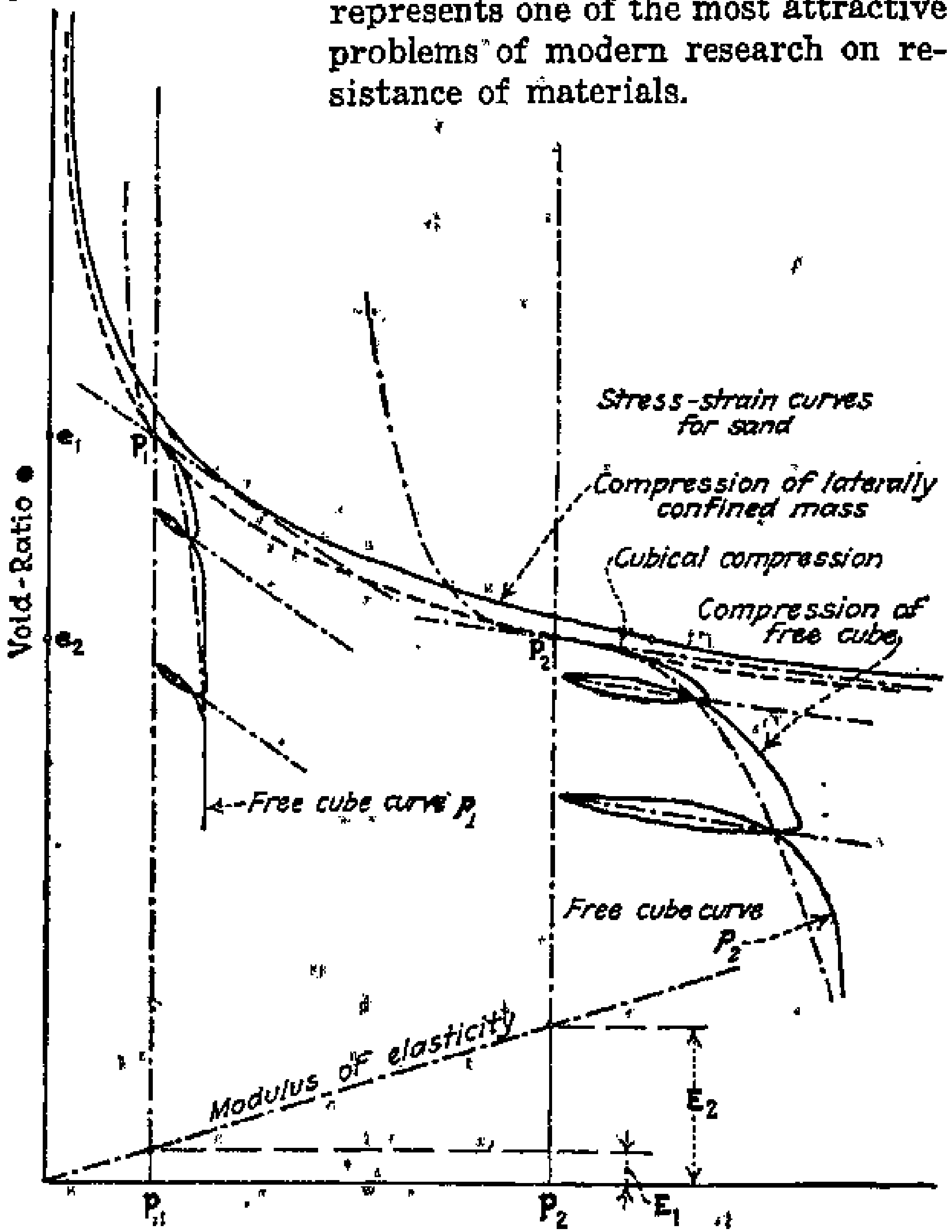

FIG. 4—STRESS-STRAIN DIAGRAM OF SAND

Due to the high ratio between modulus of elasticity and internal pressure for sands (from 238 to 419 for sands; against 31 for fat clays and about 10 for metals), a very small lateral expansion of the sand causes a considerable decrease of the lateral sand pressure. For this reason it is difficult to determine the value of earth pressure at rest, except by an indirect method such as used by the author (see "Old Earth-Pressure Theories," *Engineering News-Record*, Sept. 20, 1920, p. 632). Retaining-wall tests made by the hinged-gate method, on the other hand, are almost unavoidably affected by error because of the effect of lateral expansion.

SAND AND CLAY COMPARED AS TO PHYSICAL PHENOMENA AND THEIR CAUSES

Cause	Physical Factors Affected	Visible Consequence
Grain size.	Aggregate initial friction per unit of volume.	(1) Difference in maximum volume of voids: About 50 per cent for sand, about 98 per cent for clay.
	Capillary pressure and surface tension of the capillary water.	(2) Difference in shrinkage: Sand does not shrink in drying; clay shrinks.
		(3) Difference in cohesion: Clean sand is devoid of cohesion, clay has high cohesion.
Grain size and shape (grains bulky or scale-like).	Capillary pressure: nature of intergranular movements during deformation of the mass.	(4) Difference in plasticity: Sand is not plastic, clay is very plastic.
	Degree of permeability.	(5) Difference in speed of adjustment to loads: Sand when loaded settles to its final volume almost at once, while settlement of clay foundations proceeds very slowly.
Character of grains.	Flexibility of the particles.	(6) Difference in compressibility of the aggregate: Sand is far less compressible than clay.

Conclusions—The results of our comparison between the physical properties of sand and of clay are presented in the table. The facts stated in the first and last column of the table have long been known, but knowledge of the causative connection however between these two groups of facts is new. It is a much simpler connection than might have been expected. This table, the curves of Fig. 4, and the partial differential equation

$$\frac{K}{a} \cdot \frac{\delta^2 w}{\delta z^2} = \frac{\delta w}{\delta t}$$

represent the fundamental principles of soil mechanics in a nutshell. They are the key to the physical explanation of whatever properties a soil may display, in the laboratory or in the field.