Principles of Soil Mechanics:
VII—Friction in Sand and in Clay

Complex Nature of Friction in Granular Masses as Compared with Solid Friction—Measuring Friction in Sand and Clay—Two Kinds of Frictional Motion in Sand—Hydrodynamic Effect on Clay Friction

By Dr. Charles Terzaghi

Professor of Civil Engineering, American Robert College, Constantinople, Turkey
Temporary Lecturer, Massachusetts Institute of Technology, Cambridge

SOIL movements are largely determined by the resistance of the soil to sliding motions within the mass. Soil friction, therefore, is of peculiar importance in soil mechanics.

It is common, and quite natural, to approach the study of friction in sand and clay by way of conceptions derived from current views on friction between solids. This line of approach is apt to lead into error, however. Granular friction is radically different from solid friction in almost every phase, and the two subjects require separate and quite distinct modes of attack.

Largely through the work of W. B. Hardy and collaborators in very recent years, remarkable insight into the phenomena of solid friction has been gained, and the action is now known to be far more complex than had been thought during the last century or two. The coefficient of friction between perfectly clean and smooth surfaces was found to be very high and constant. However, in practice no perfectly clean surfaces exist because contaminating matter spreads over every surface either by creep or by condensation and the presence of contaminating matter considerably reduces the value of the coefficient of friction. Water is called by Hardy an anti-lubricant because it has the property of increasing the value of the coefficient of friction between contaminated, smooth surfaces by partially compensating the lubricating effect of contamination. In no case was it found to act as a lubricant. Among lubricating oils of similar chemical character the power of reducing the coefficient of friction increases with increasing molecular weight. For uneven surfaces the coefficient of friction depends in addition on whether or not the pressure acting per unit of area of the surface of actual contact is high enough to permit the minute projections to cut through the lubricating film and produce incipient abrasion.

Although it thus involves a number of factors, solid friction is nevertheless a relatively simple phenomenon, from the physical point of view. The contacting bodies are constrained to move parallel to their interface, and conditions remain fairly constant as the motion progresses, provided the motion is very slow.

In contrast, motion between masses of individual grains is a very complex process, involving not merely surface friction but also translation and rotation of the grains. The complexity of the process betrays itself in the fact that the actual movement is preceded by an incipient one, caused by minor grain displacements. In short, frictional resistance in sand depends not only on the pressure and the nature of the contacting surfaces but also on the density of structure of the sand and on the thickness of the zone to which the grain movements are confined.

Internal Friction of Sand—Extensive experiments were made by the Foundation Soils committee of the American Society of Civil Engineers to determine the coefficient of friction of Ottawa standard sand (see *Transactions*, Am. Soc. C. E., 1917, 1920), but no simple and general conclusions have resulted thus far. A rather elaborate disk apparatus was used, in which the sand was confined within radially arranged compartments. The coefficient of friction was computed from the force required to produce a rotation of the mobile part of the apparatus. The coefficient of internal friction thus obtained was remarkably low, but varied considerably with the number of preceding runs. The coefficient for the friction between standard sand and the surface of standard sand mortar was found to be considerably higher than the coefficient for sand on sand. In each series of tests the coefficient sensibly decreased (at low and at medium pressure) with increasing pressure. The results show that the coefficient of friction even of standard sand is likely to vary between surprisingly wide limits, depending on the test conditions.

In fact, there is no definite single coefficient of friction. The value of the coefficient depends, both on the manner in which slip is produced and on the changes in structure of the sand prior to the slip. On theoretical considerations as well as by the results of experimental work, the author has been led to distinguish between two fundamentally different cases:

(a) Separation of a mass of sand along a plane: Within the zone of slip there is complete rearrangement of the grains. Since Reynolds' classical experiment with the sand-bag we know that the grains of a mass of sand cannot change partners unless the volume of voids of the sand temporarily increases. Hence the separation along a plane requires a gradual loosening up of the structure of the sand in the vicinity of the plane of separation, i.e., the separation involves a tendency toward increasing the volume of voids. The coefficient of friction along a plane of separation is called the *coefficient of internal friction.*

(b) Continuous deformation of a mass of sand: If a mass of sand is uniformly stretched or compressed so that the displacement of the grains occurs throughout the whole mass, the stability of the structure increases and finally assumes a maximum. In this limiting stage the frictional resistance which determines the ratio between the extreme principal stresses is the *coefficient of internal resistance.* Its value represents the maximum value which the coefficient of friction can assume within a mass of sand of a given density. It may be considerably greater than the slope of repose.

Cases (a) and (b) are limiting cases; there is an infinite number of intermediate possibilities, each of which corresponds to another type of internal displacement, involving another coefficient of friction, limiting the state of equilibrium for that particular type of displacement. Thus the value of the coefficient of internal friction depends on whether the slip is confined to a zone only a few grains in thickness or whether it occurs within a layer ten times as thick. This seems to be the chief reason for the disagreement between the results of large-scale retaining-wall tests and miniature tests.

The friction tests of the Foundation Soils committee are a notable example of the utter complexity of the frictional resistance in sands. It is timely to conclude from these and similar experiments that there can be no hope of exactly determining the value of the coefficient of friction required for calculating the outcome of more complicated earth pressure phenomena in sands. We can only guess at that value by choosing some intermediate value between the extreme ones or by indirectly computing it from what has been observed.

Internal Friction in Clay—In the case of sand, then, popular conceptions of the nature of frictional resistance are widely at variance with the physical facts. The same thing is even more true for clay.

Be it stated in advance: Experiments to determine the coefficient of internal friction of clay must (a) exclude the surface tension of the capillary water, because this tension produces internal pressures in addition to the pressure applied externally, and (b) take account of the fact that the frictional resistance depends not only on the intensity of the pressure but (within limits) also on the length of time during which the pressure acts.

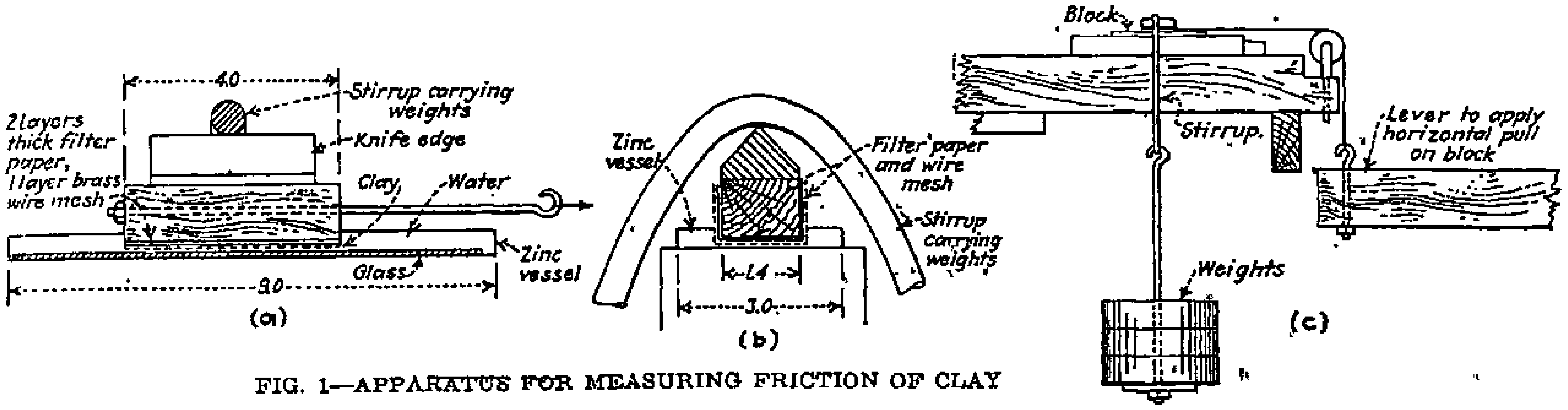

FIG. 1—APPARATUS FOR MEASURING FRICTION OF CLAY

Suppose the plane surface of a mass of air-free wet clay is under a definite external pressure. When the pressure is increased the clay body becomes smaller, like any other elastic body. As the voids of the clay are filled with water, the increase in pressure involves escape of the excess water. But as the permeability of the clay is very small, the escape of the water proceeds very slowly. During the drainage process part of the excess pressure is compensated for by a corresponding hydrostatic pressure of the water in the voids; the surcharge partly floats on the surface, so to say, and only the surplus is carried by the solid parts of the mass. Immediately after the surcharge is applied, the compression of the clay is practically equal to zero, hence the hydrostatic pressure at this time is almost equal to the surcharge. While the excess water gradually escapes, the hydrostatic pressure becomes smaller and approaches the value zero. The speed of decrease evidently depends both on the coefficient of permeability of the clay and on the thickness of the clay layer. A hydrostatic pressure does not produce any static friction. Hence it is merely the remainder of the surcharge which counts. The frictional resistance increases with decreasing hydrostatic pressure, and does not assume its normal value until the excess water has completely escaped from the layer of clay.

Herein resides the physical cause of the slipperiness of the surface of fat clays. If one steps suddenly on a very slightly inclined clay-surface the foot slips, although even a very fat clay has a fairly large internal friction (friction angle at least 11 deg.). In the rapid application of the pressure of the foot, the greatest part of the weight of the body is compensated for by hydrostatic pressure, and the friction produced by the remaining weight is not sufficient to prevent the slip.

Certain types of landslides involve a similar phenomenon. The slide is preceded by the formation of shrinkage cracks or other open spaces, in which water accumulates. Due to some spontaneous displacement in the fissured material part of the weight acting in the trapped layer of water compensates part of the weight of the settling material with a speed corresponding to almost zero friction on the sliding plane. This phenomenon, in which the coefficient of friction may temporarily go down to 0.05 or less, is commonly accounted for by the lubricating action of the water. In fact it is merely due to a spontaneous partial compensation of the weight of the overburden by the hydrostatic excess pressure acting in a layer of trapped water.

Friction Apparatus—In order to fulfill the requirements essential for a successful test, the author investigated the internal friction of clays by means of the apparatus represented in Fig. 1. A pan of zinc has its bottom covered with a sheet of glass. For determining the coefficient of friction between glass and clay, the glass sheet is covered with a thin layer of plastic mixture of clay and water (free of air). On the layer of clay is placed a prismatic block whose base and sides are lined with two layers of thick filter paper and one layer of No. 20 brass wire mesh. The brass mesh acts as a rough surface, which takes the clay with it, and the filter paper serves as a drain for the excess water. Load is applied by weights hung from a stirrup resting on a knife-edge above the loading block. The load usually reduced the thickness of the layer of clay to about 3 mm. Finally all the clay fragments projecting beyond the edges of the base of the loading block were scraped away and the zinc vessel was filled with water. Horizontal pull was applied to overcome the frictional resistance, by means of a cord leading over a pulley to the end of a loading lever.

The friction tests were never started earlier than 24 hours after the load was applied, a time sufficient for draining the thin layer of clay or allowing it to become resaturated. When it was desired to measure the coefficient of frictional resistance of clay on clay, the glass plate was replaced by a double layer of filter paper covered with a brass wire mesh.

This test arrangement is far from being ideal, because the tests require much care, time and experience. However, for the time being it is the only one which fulfills the above-mentioned first condition for a successful test. Further efforts will be required for developing a more convenient and perfect device.

Friction Test Results—The results showed the coefficient of friction of clay to be remarkably constant for pressures of more than about 1 kg./cm.², and to be remarkably independent of many of the factors which distinctly influence the coefficient of friction of sands. This seems to be due to the fact that fat clays consist of thin and very flexible flakes, while sand is composed of bulky, fairly rigid grains. The structure of sand does not allow change of shape without change in volume of voids (sandbag experiment of Reynolds), while clay is capable of plastic deformation (deformation at a constant volume of voids).

The coefficient of friction between glass and clay is about 0.18 to 0.22 for colloidal mud, 0.23 to 0.30 for fat clays, and 0.30 to 0.32 for sandy clay. The respective coefficients of internal friction of the clays are 0.23 to 0.28, 0.25 to 0.40, and 0.40 to 0.50. The coefficients of internal friction and of internal resistance seem to be almost identical. Hence each kind of clay has its definite coefficient of internal friction. Herein lies an essential difference between sand and clay.

The sliding of clay on the smooth surface of the glass sheet occurred suddenly, as soon as the pull exceeded a threshold value. Motion along an interface located within the clay, on the other hand, proceeded rather gently, resembling somewhat the flow in a very viscous liquid. The initial friction, corresponding to zero pressure, amounted to about 20 g./cm.² The smaller the pressure, the greater was the influence of initial friction on the value of the coefficient, as may be seen from the high values at the left-hand end of curves, Fig. 2.

In each test, actual motion was preceded by an incipient motion, usually starting as soon as the pull became equal to about six-tenths of the threshold value. The incipient motion seems to be caused by the structure of the stretched clay adapting itself to the changing state of stress. If the pull was left constant before it had attained the threshold value, the incipient motion continued with decreasing speed and ceased completely within twenty-four hours. After this a further increase of pull produced no detectable incipient motion whatever.

For clays the coefficients of internal friction and of internal resistance seem to be almost identical. In a preceding paper I have shown that the shearing strength of a plastic sample of clay is equal to the product of capillary pressure times the coefficient of internal resistance. Hence there is a contradiction between the low coefficient of internal resistance and the great cohesion of certain fat clays, explained by the great intensity of the stress acting in the capillary water of these clays.

The Foundation Soils committee, in investigating the internal friction of clays, used a device similar to the one described for sand tests at the opening of this article. But the essential requirements for obtaining normal values for the coefficient of internal friction were not fulfilled. The Kentucky ball clay with 10 per cent and 25 per cent water, respectively, undoubtedly contained considerable quantities of air, or else the cohesion effect would have been far greater than that actually measured. The coefficient of friction was of the order 0.25 (0.23 to 0.27). The tests made with a mixture of clay with 39.54 per cent of water furnished an exceedingly low value of friction coefficient which clearly proves that the greatest part of the surcharge was compensated for by hydrostatic pressure, i.e., the test was made a long time before the excess water had completely escaped. In a test of such a kind the clay is not yet in hydrostatic equilibrium, and the test may furnish any friction value between zero and the normal value. Such values are not the coefficients of static friction, but coefficients of what may be called the momentary hydrodynamic friction; they depend on the pressure which acted on the clay prior to the time when the surcharge was applied, on the time during which the surcharge was allowed to act, on the thickness of the layer, and on various other factors.

Conclusions—The quantitative side of every earth-pressure phenomenon depends on the intensity of the frictional resistance acting within the soil. Through decades of years the development of soil mechanics has been handicapped by the traditional habit of identifying the laws which govern the frictional resistance in granular masses with those derived from friction tests performed with solid bodies. In the preceding the attempt has been made to characterize the phenomena of sand and clay friction and to indicate their quantitative features.

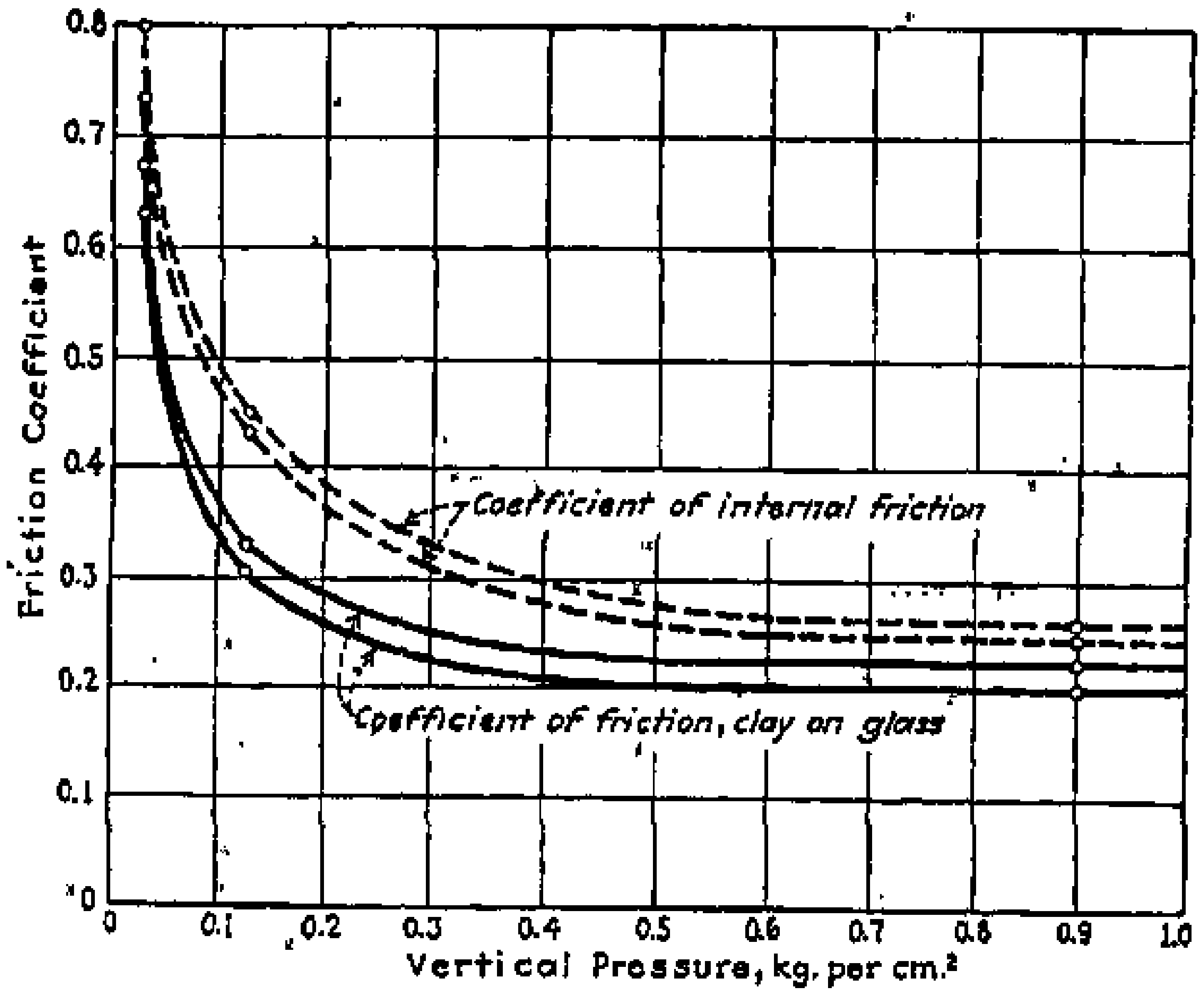

FIG. 2—STATIC FRICTION OF YELLOW CLAY

On the basis of these considerations taken in conjunction with the elaborate studies of Hardy and others on friction between solids with and without lubricants, the following statements may be made:

(*A*) Friction between smooth and absolutely clean surfaces of solid bodies is a purely physico-chemical process and is caused by direct molecular interaction.

(*B*) Friction between imperfectly smooth surfaces of solid bodies involves not only these physico-chemical causes but also a file-like action of each surface on the other. Nevertheless, from a physical point of view the phenomenon is a simple one.

(*C*) In sand the friction coefficient depends not only on the properties of the grains and the structure of the sand, but also on the nature of the process which causes the slip, and on the nature of the process which preceded the slip. It has no definite value, but may be anywhere between two limiting values, (a) the coefficient of internal friction (friction acting along a plane of separation), and (b) the coefficient of internal resistance (resistance which develops while the entire mass suffers a uniform deformation).

(*D*) In clay the friction coefficient for medium and for high pressures is remarkably constant. For low pressures, however, the value of the coefficient increases with decreasing pressure, because initial friction plays an important part, amounting to about 20 g./cm.^{2} Rapid change of pressure produces in the liquid component of the clay a positive or a negative hydrostatic pressure. The coefficient of friction does not assume its normal value (coefficient of static friction) until the hydrostatic pressure has become zero throughout the whole mass. In the preceding stage of the process the coefficient of friction (coefficient of hydrodynamic friction) may have any positive value, and is a function of the time.